Examples from class using R for log-log plots

Example 1 **Surface area of a sphere as a function of its volume example.**

(a) First examine the surface area of a sphere as a function of the volume. Let r be the radius, S the surface area, and V the volume. We know that

$$S = 4\pi r^2$$
$$V = \frac{4\pi}{3} r^3.$$

To find S as a function of V first solve for V as a function of r:

$$r = \left(\frac{3}{4\pi} V\right)^{1/3},$$

and then plug this into the formula for S to get

$$S = 4\pi r^2 = 4\pi \left(\frac{3}{4\pi} V\right)^{2/3} = (4\pi)^{1/3} 3^{2/3} V^{2/3} = (36\pi)^{1/3} V^{2/3}$$

(b) Use R to make a plot of this

```
r=seq(from=0.1,to=10,by=0.1)
S=4*pi*r^2
V=4*pi/3*r^3
plot(V,S,type="l",xlab="volume",ylab="surface area")
```

(c) Make a log plot. First define new variables which are the log of the original data, and then plot.

```
LS=log10(S)
LV=log10(V)
plot(LV,LS,type="l",xlab="log(volume)",ylab="log(surface area)")
```

(d) We expect a line of slope 2/3 and intercept $\log_{10}(36\pi)^{1/3} \approx 0.6845$. We can fit a line to the data to verify this using the below command.

```
F=lm(LS~LV)
F
```

The output from these command is

```
Call:
lm(formula = LS ~ LV)

Coefficients:
(Intercept)           LV
     0.6845       0.6667
```

So, indeed this line has the slope and intercept we predicted above.

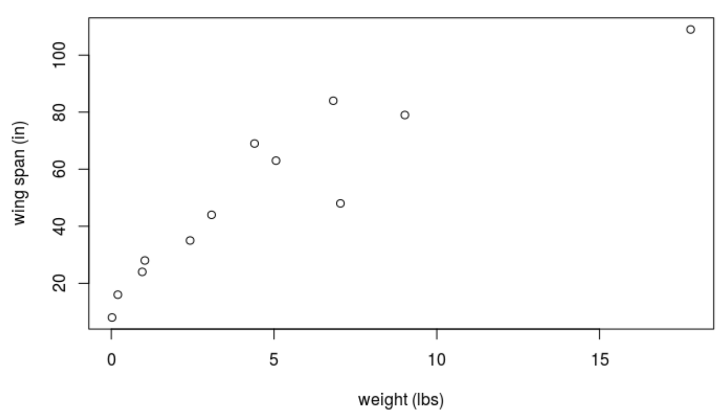
Example 2 **Estimating the wingspan of flying humans.**

Below is a table with the weight and wingspan for different flying birds. We will use this data to find a model for how the wingspan depends on the weight, and then use the model to predict the wingspan of a flying human.

Bird	Avg. body weight (W, lb)	Avg. wingspan (L, in)
Turkey vulture	4.40	69
Bald eagle	6.82	84
Great horned owl	3.08	44
Cooper's hawk	1.03	28
Sandhill crane	9.02	79
Atlantic puffin	0.95	24
California condor	17.8	109
Common loon	7.04	48
Yellow warbler	0.022	8
Common grackle	0.20	16
Wood stork	5.06	63
Mallard	2.42	35

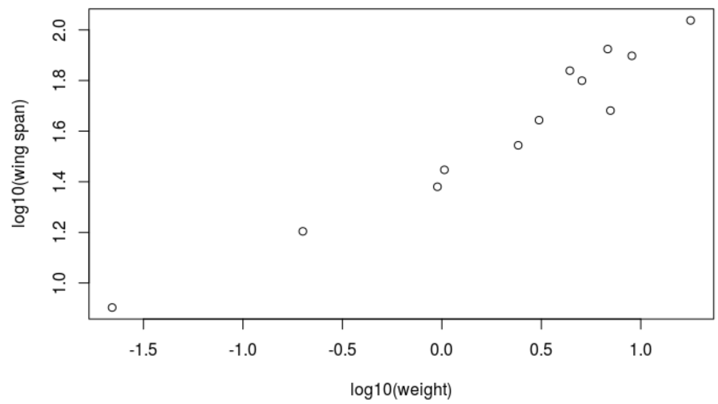
(a) Enter the data and make a plot the wingspan vs. weight using the commands below.

```
wgt=c(4.4,6.82,3.08,1.03,9.02,0.95,17.8,7.04,0.022,0.2,5.06,2.42)
wspan=c(69,84,44,28,79,24,109,48,8,16,63,35)
plot(wgt,wspan,xlab="weight (lbs)",ylab="wing span (in)")
```



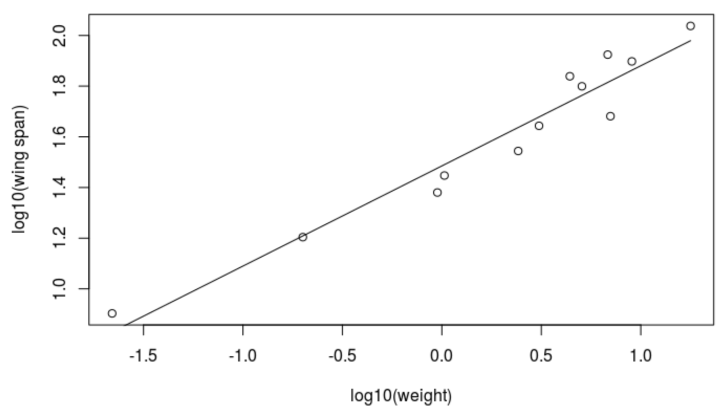
(b) Take the log of the data and replot.

```
Lwgt = log10(wgt)
Lwspan = log10(wspan)
plot(Lwgt,Lwspan,xlab="log10(weight)",ylab="log10(wing span)")
```



(c) Find the best fit line, and add it to the graph

```
F=lm(Lwspan~Lwgt)
lines(Lwgt,fitted(F))
```



(d) Identify the parameters of the fit, and find the equation.

```
F
```

The output from this command is below

```
Call:
lm(formula = Lwspan ~ Lwgt)

Coefficients:
(Intercept)         Lwgt
     1.4852       0.3952
```

This means that the slope of the best fit line is approximately $m = 0.3952$ and the intercept is approximately $b = 1.4852$.

(e) Identify the equation that relates wing span to weight based on the best fit line on the log-log graph. The equation of the best fit line is

$$\log_{10}(wspan) = m\log_{10}(wgt) + b.$$

To transform back to original variables, exponentiate both sides with base 10.

$$\begin{aligned} 10^{\log_{10}(wspan)} &= 10^{m\log_{10}(wgt)+b} \\ wspan &= 10^{\log_{10}(wgt)^m}10^b \\ wspan &= 10^b wgt^m \end{aligned}$$

Using the above values for b and m, we get

$$wspan = 30.56\, wgt^{0.3952}.$$

(f) According the the model above, what would be the wing span of a 200 pound human?

$$30.56\,(200)^{0.3952} \approx 248$$

The wingspan of a 200 pound human would be about 248 inches (20.67 feet, or 6.3 meters).