Math 207C
Homework 7
Due Friday, June 1st

1. Find the first term approximation valid for long time to the initial value problem

$$\ddot{u} + \epsilon(u^2 - 1)\dot{u} + u = 0$$
$$u(0) = 0, \quad \dot{u}(0) = 1.$$

2. (a) A regular expansion of the solution to

$$y'' + y + \epsilon\beta y' + \epsilon\alpha y^3 = F\cos(\Omega t)$$

as $\epsilon \to 0$ will lead to secular terms in either the order one or the order ϵ equation. Find the values of Ω so that the forcing term contributes to the secular terms.

(b) The leading order solution to the above equation has the form

$$y = A(t)\cos(t + \phi(t)) + C\cos(\Omega t) + \mathcal{O}(\epsilon).$$

Using the method of multiple scales, derive a differential equation for A and an expression for C for Ω away from the frequencies identified in the previous part. Comment on the effect of the nonlinearity of this result. That is, what does the nonlinearity affect in leading order solution at nonresonant frequencies?

Note: the textbook analyzes the case where the forcing frequency is near the dominant resonance. When the frequency is near a resonant frequency, the nonlinarity has a substantially different effect on the solution.