1. A common feature of non-Newtonian fluids is shear thinning. A simple model that exhibits shear thinning results from the assumption that the viscous stress is a nonlinear function of the deformation rate. For example, for the shear flow \( \mathbf{u} = (u(y), 0, 0) \), the viscous stress in a shear thinning power-law fluid is \( \sigma_{12} = \mu_0 |u_y|^{-\alpha} u_y \) for \( 0 \leq \alpha < 1 \).

(a) Find the velocity profile of the steady flow between stationary parallel plates at constant pressure gradient. Hint: Consider the flow in half of the channel with the symmetry condition at the center \( u_y = 0 \), and assume that \( u_y \) does not change sign in the half-channel.

(b) Plot the velocity profiles (normalized by the max velocity) for several choices of \( \alpha \). Although this model is phenomenological, describe the general effect of shear thinning on the velocity profile in a pressure-driven channel flow.

2. Consider the steady flow between two concentric rotating cylinders. The inside cylinder has radius \( R_{in} \) and rotates with angular velocity \( \Omega_{in} \). The outside cylinder has radius \( R_{out} \) and rotates with angular velocity \( \Omega_{out} \). This is called a Couette flow.

(a) Compute the velocity field, and plot the velocity profile.

(b) Compute the pressure.

(c) Find the shape of the free surface on top of an infinite bath in which a rod is rotating at a constant angular velocity.

3. Suppose that initially the velocity is the point vortex \( \mathbf{u} = \frac{\Gamma_0}{2\pi r} e_\theta \). This is a steady solution for an ideal fluid, but in a viscous fluid this flow will not persist.

(a) Solve for the velocity field as a function of time for a Newtonian fluid. Assume that the only nonzero component of the velocity is in the \( \theta \) direction and that it is a function of time and \( r \) only.

(b) Note that the initial flow is irrotational for \( r > 0 \). Compute the vorticity as a function of time. What does this solution suggest about the initial vorticity?