1. Do the following sequences \( \{a_n\} \) converge or diverge as \( n \to \infty \)? If a sequence converges, find its limit. Justify your answers.

(a) \( a_n = \frac{2n^2 + 3n^3}{2n^3 + 3n^2} \);  
(b) \( a_n = \cos(n\pi) \);  
(c) \( a_n = \frac{\sin(n^2)}{n^2} \).

2. Do the following series converge or diverge? State clearly which test you use.

(a) \( \sum_{n=1}^{\infty} \frac{n + 4}{6n - 17} \)

(b) \( \sum_{n=2}^{\infty} \sqrt{\frac{n}{n^4 + 7}} \)

(c) \( \sum_{n=1}^{\infty} \frac{(-5)^{n+1}}{(2n)!} \)

(d) \( \sum_{n=3}^{\infty} \frac{\ln n}{n} \)

(e) \( \frac{1}{1^4} + \frac{1}{2^4} - \frac{1}{3^4} + \frac{1}{4^4} + \frac{1}{5^4} - \frac{1}{6^4} + \frac{1}{7^4} - \frac{1}{9^4} + \cdots \)

(f) \( \sum_{n=1}^{\infty} [e^n - e^{n+1}] \)

3. Determine the interval of convergence (including the endpoints) for the following power series. State explicitly for what values of \( x \) the series converges absolutely, converges conditionally, and diverges. Specify the radius of convergence \( R \) and the center of the interval of convergence \( a \).

\[ \sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n} (x - 1)^n. \]

4. Write the Taylor polynomial \( P_2(x) \) at \( x = 0 \) of order 2 for the function \( f(x) = \ln(1 + x) \).
Use Taylor’s theorem with remainder to give a numerical estimate of the maximum error in approximating \( \ln(1.1) \) by \( P_2(0.1) \).

5. (a) Find the value(s) of \( c \) for which the vectors
\[
\vec{u} = c\vec{i} + \vec{j} + c\vec{k}, \quad \vec{v} = 2\vec{i} - 3\vec{j} + c\vec{k}.
\]
are orthogonal.
(b) Find the value(s) of \( c \) for which the vectors
\[
\vec{u} = c\vec{i} + \vec{j} + c\vec{k}, \quad \vec{v} = 2\vec{i} - 3\vec{j} + c\vec{k}, \quad \vec{w} = \vec{i} + 6\vec{k}.
\]
lie in the same plane.

6. Find a parametric equation for the line in which the planes
\[
3x - 6y - 4z = 15 \quad \text{and} \quad 6x + y - 2z = 5
\]
intersect.

7. Suppose that
\[
f(x, y) = e^x \cos \pi y
\]
and
\[
x = u^2 - v^2, \quad y = u^2 + v^2.
\]
Using the chain rule, compute the values of
\[
\frac{\partial f}{\partial u}, \quad \frac{\partial f}{\partial v}
\]
at the point \((u, v) = (1, 1)\).

8. Let
\[
f(x, y, z) = \ln \left(x^2 + y^2 - 1\right) + y + 6z.
\]
In what direction \( \vec{u} \) is \( f(x, y, z) \) increasing most rapidly at the point \((1, 1, 0)\)? Give your answer as a unit vector \( \vec{u} \). What is the directional derivative of \( f \) in the direction \( \vec{u} \)?

9. Find the equation of the tangent plane to the surface
\[
xyz = 2
\]
at the point $(1, 1, 2)$.

10. Find all critical points of the function

$$f(x, y) = x^4 - 8x^2 + 3y^2 - 6y.$$  

and classify them as maximums, minimums, or saddle-point.

11. Let

$$D = \{(x, y) : x^2 + y^2 \leq 1\}$$  

be the unit disc and

$$f(x, y) = x^2 - 2x + y^2 + 2y + 1.$$  

Find the global maximum and minimum of

$$f : D \to \mathbb{R}$$  

At what points $(x, y)$ in $D$ does $f$ attain its maximum and minimum?

12. Suppose that the material for the top and bottom of a rectangular box costs $a$ dollars per square meter and the material for the four sides costs $b$ dollars per square meter. Use the method of Lagrange multipliers to find the dimensions of a box of volume $V$ cubic meters that minimizes the cost of the materials used to construct it. What is the minimal cost?