As illustrated in Figure 1, we choose two constant vectors \( a, b \in \mathbb{R}^3 \) that are linearly independent from \( \gamma'(t_0) \), which is possible since \( \gamma'(t_0) \neq 0 \). (For example, we can use the normal and binormal vectors to the curve \( \gamma \) at \( t_0 \).) We then define \( F : I \times \mathbb{R}^2 \rightarrow \mathbb{R}^3 \) by

\[
F(t, u, v) = \gamma(t) + ua + vb.
\]

That is, \( F(t, u, v) \) is the point in \( \mathbb{R}^3 \) that is obtained by going \( u \) in the \( a \)-direction and \( v \) in the \( b \)-direction from \( \gamma(t) \). In particular, \( F(t, 0, 0) = \gamma(t) \).

The function \( F \) is \( C^1 \) since \( \gamma \) is \( C^1 \) and

\[
\frac{\partial F}{\partial t}(t, u, v) = \gamma'(t), \quad \frac{\partial F}{\partial u}(t, u, v) = a, \quad \frac{\partial F}{\partial v}(t, u, v) = b.
\]

Moreover, the differential matrix of \( F \) is

\[
[dF(t, u, v)] = \begin{bmatrix}
\gamma'(t) & a & b
\end{bmatrix},
\]

where \( \gamma'(t), a, b \) are interpreted as column vectors. The differential \( dF(t_0, 0, 0) \) has rank 3 and is invertible, since \( \{\gamma'(t_0), a, b\} \) are linearly independent.

The inverse function theorem implies that there exist neighborhoods \( U \) of \( (t_0, 0, 0) \) and \( V \) of \( \gamma(t_0) \) such that \( F : U \rightarrow V \) has a \( C^1 \)-inverse \( F^{-1} : V \rightarrow U \). We write \( F^{-1} = (e, f, g) \) where \( e, f, g : V \rightarrow \mathbb{R} \) are \( C^1 \) and

\[
t = e(x, y, z), \quad u = f(x, y, z), \quad v = g(x, y, z).
\]

If \( (x, y, z) \in V \), then \( (x, y, z) = \gamma(t) \) if and only if \( F^{-1}(x, y, z) = (t, 0, 0) \). Thus, in \( V \), the curve \( \gamma \) is the solution of the equations

\[
f(x, y, z) = 0, \quad g(x, y, z) = 0.
\]

Geometrically, the curve is the intersection of the two surfaces \( f(x, y, z) = 0 \) and \( g(x, y, z) = 0 \).
Figure 1: The curve $\gamma$ and the map $F$. 