1. Suppose that $f_n, f : [a, b] \to \mathbb{R}$ are integrable functions and $f_n \to f$ as $n \to \infty$ uniformly on $[a, b]$. Let

$$F_n(x) = \int_a^x f_n(t) \, dt, \quad F(x) = \int_a^x f(t) \, dt.$$ 

Prove that $F_n \to F$ uniformly on $[a, b]$. If $f_n \to f$ pointwise, does it follow that $F_n \to F$ pointwise?

2. Let $f_n : (a, b) \to \mathbb{R}$ be a sequence of differentiable functions whose derivatives $f'_n : (a, b) \to \mathbb{R}$ are integrable on $(a, b)$. Suppose that $f_n \to f$ pointwise and $f'_n \to g$ uniformly on $(a, b)$ as $n \to \infty$, where $f, g : (a, b) \to \mathbb{R}$ and $g$ is continuous. Prove that $f$ is differentiable in $(a, b)$ with $f' = g$. Where do you need to use the continuity of $g$?

**HINT.** Show that the indefinite integrals of $f'_n$ converge and use the fundamental theorem of calculus.