1. Say if the following statements are true or false. If true, give a brief explanation (a complete proof is not required); if false, give a counterexample.
   (a) If \( A, B \subset \mathbb{R} \) are nonempty sets that are bounded from above and \( A \cap B \) is nonempty, then
       \[
       \sup(A \cap B) = \min\{\sup A, \sup B\}. 
       \]
   (b) If \( A, B \subset \mathbb{R} \) are nonempty sets that are bounded from above, then
       \[
       \sup(A \cup B) = \max\{\sup A, \sup B\}. 
       \]
   (c) If \((x_n)\) is a convergent sequence and \((y_n)\) is a sequence such that \( y_n \to \infty \) as \( n \to \infty \), then there exists \( N \in \mathbb{N} \) such that \( y_n > x_n \) for every \( n > N \).
   (d) If a sequence is bounded from above, then it has a convergent subsequence.

2. Suppose that \( x \geq 0 \). Prove by induction that
   \[
   1 + nx \leq (1 + x)^n \text{ for every } n \in \mathbb{N}. 
   \]

3. (a) Let \( A \subset \mathbb{R} \) be a nonempty set that is bounded from above. Prove that there is a sequence \((a_n)\) of points \( a_n \in A \) such that \( a_n \to \sup A \) as \( n \to \infty \).
   (b) Can the sequence in (a) be chosen so that it is increasing? Briefly explain your answer (a complete proof is not required).

4. (a) State the definition of a Cauchy sequence \((x_n)\) of real numbers.
   (b) Let \( x_n = \sqrt{n} \). Prove that for every \( \epsilon > 0 \), there exists \( N \in \mathbb{N} \) such that
       \[
       |x_{n+1} - x_n| < \epsilon \text{ for every } n > N. 
       \]
   (c) Is \((x_n)\) a Cauchy sequence?

5. (a) Let \((x_n)\) be a sequence of real numbers and \( x \in \mathbb{R} \). Define \( y_n = |x_n - x| \). Prove that the sequence \((x_n)\) is bounded if and only if the sequence \((y_n)\) is bounded.
   (b) Prove that \( \lim_{n \to \infty} x_n = x \) if and only if \( \limsup_{n \to \infty} |x_n - x| = 0 \).