1. Say if the following statements are true or false. If true, give a brief explanation (a complete proof is not required); if false, give a counterexample.
(a) If \( \sum a_n \) diverges and \( 0 \leq a_n \leq b_n \), then \( \sum b_n \) diverges.
(b) If the series \( \sum a_n \) is conditionally convergent, then the series \( \sum \sqrt{n}a_n \) diverges.
(c) If \( A \subset \mathbb{R} \) has closure \( \bar{A} = \mathbb{R} \), then \( A \) is uncountable.
(d) If \( A \subset \mathbb{R} \) and every \( a \in A \) is an isolated point of \( A \), then \( A \) is closed.

2. (a) State the Cauchy condition for the convergence of a series \( \sum_{n=1}^{\infty} a_n \).
(b) Suppose that \( (a_n)_{n=1}^{\infty} \) is a sequence of real numbers and \( (a_{n_k})_{k=1}^{\infty} \) is a subsequence. If the series \( \sum_{n=1}^{\infty} a_n \) converges absolutely, prove that the series \( \sum_{k=1}^{\infty} a_{n_k} \) converges.
(c) Does the result in (b) remain true if \( \sum_{n=1}^{\infty} a_n \) converges conditionally? Justify your answer.

3. Determine the convergence of the following series and justify your answers (you can use any test):
\[
\begin{align*}
(a) \quad & \sum_{n=1}^{\infty} \frac{2^n}{\sqrt{n!}}; \\
(b) \quad & \sum_{n=1}^{\infty} \frac{n - 1}{n^2 + 1}; \\
(c) \quad & \sum_{n=1}^{\infty} \left( \frac{n - 1}{n} - \frac{n}{n + 1} \right).
\end{align*}
\]

4. (a) Define an open set \( G \subset \mathbb{R} \).
(b) Let \( A \subset \mathbb{R} \) and \( \epsilon > 0 \). Prove that the following set is open:
\[
G = \{ x \in \mathbb{R} : |x - a| < \epsilon \text{ for some } a \in A \}.
\]

5. Suppose that \( A \subset \mathbb{R} \) is nonempty and closed, and \( b \in \mathbb{R} \). Prove that there exists \( a \in A \) such that \( |a - b| = \inf \{ |x - b| : x \in A \} \). Is \( a \) unique?