MAT 127B-01 Winter 05 Midterm 1
Professor Art Krener

1.(20 pts) Consider the power series

$$\sum_{n=1}^{\infty} \frac{3^n}{n} x^{2n}$$

a. Compute its radius R of convergence.
b. Does it converge at $x = \pm R$, explain your answer.

2.(20 pts) Consider the sequence of functions $f_n(x) = x - x^n$.
a. Does this sequence converge pointwise on $[0,1]$? If so what is the limit function?
b. Does this sequence converge uniformly on $[0,1]$? Prove your answer.
c. Does this sequence converge uniformly on $[0,a]$ where $0 < a < 1$? Prove your answer.

3. (20 pts) Show that $\sum_{n=1}^{\infty} \frac{\sin nx}{n^3}$ converges to a continuous function on $(-\infty, \infty)$.

4.(20 pts) Suppose $f_n(x)$ converges uniformly on $[a,b]$ to a bounded function $f(x)$ and $g_n(x)$ converges uniformly on $[a,b]$ to a bounded function $g(x)$. Show that $h_n(x) = f_n(x)g_n(x)$ converges uniformly to $h(x) = f(x)g(x)$.

5.(20 pts) Find the power series of the function

$$f(x) = \sum_{n=0}^{\infty} a_n x^n$$

that satisfies the differential equation

$$f'(x) = xf(x) + 1, \qquad f(0) = 0$$