1. Show that
\[ d(x, y) = \frac{|x - y|}{1 + |x - y|} \]
is a metric on \( \mathbb{R} \). Is \( \mathbb{R} \) complete with respect to this metric?

2. Does the equation
\[ x^5 + y^5 + xy + 4 = 0 \]
define an implicit function \( x = g(y) \) locally near the point \((x, y) = (-2, 2)\)? Explain your answer.

3. Suppose that \( 1/2 \leq a \leq 3/2 \). Define a function \( \phi : \mathbb{R} \to \mathbb{R} \) by
\[ \phi(x) = x + \frac{1}{2} (a - x^2) \]
Find a closed bounded interval \( I \subset \mathbb{R} \) containing 1 such that \( \phi : I \to I \) is a contraction. If \( x_0 \in I \), what do the iterates
\[ x_{n+1} = x_n + \frac{1}{2} (a - x_n^2) \]
converge to as \( n \to \infty \)?

4. Use the change of variables formula to transform
\[ \int_0^\infty \int_0^\infty e^{-x^2-y^2} \, dx \, dy \]
into an integral with respect to polar coordinates \((r, \theta)\), where
\[ x = r \cos \theta, \quad y = r \sin \theta. \]
Deduce the value of
\[ \int_0^\infty e^{-x^2} \, dx \]
Justify your steps.