1. A sequence of bounded linear operators $A_n \in B(H)$ on a Hilbert space $H$ is said to converge to an operator $A \in B(H)$:\n\hspace{1cm} uniformly if $A_n \to A$ with respect to the operator norm on $B(H)$; strongly if $A_n x \to Ax$ strongly in $H$ for every $x \in H$; weakly if $A_n x \rightharpoonup Ax$ weakly in $H$ for every $x \in H$.

(a) Give an example of a sequence of operators that converges strongly but not uniformly.

(b) Give an example of a sequence of operators that converges weakly but not strongly.

2. A subset $E$ of a (real or complex) vector space $X$ is said to be convex if

$$\lambda x + (1 - \lambda)y \in E \quad \text{for every } x, y \in E \text{ and every } 0 \leq \lambda \leq 1.$$ 

(a) Show that a closed (i.e. strongly closed), convex subset of a Hilbert space is weakly closed.

(b) Show that every closed, convex subset of a Hilbert space contains a point of minimum norm.