1. For each of the following systems, find the equilibria and their stability. Determine what bifurcations occurs, sketch the bifurcation diagram, and sketch the qualitatively different phase lines:

(a) \( x_t = \mu - x^2 + x^4 \);  (b) \( x_t = \mu x + x^3 - x^5 \);  (c) \( x_t = \mu x - e^x \).

2. (a) Consider a pair of rigid rods of length \( L \) connected by a torsional spring with spring constant \( k \) that resists bending. If the rods are subject to a compressive force \( \lambda \), and \( x \) is the angle of the rods to the applied force, explain why

\[
V(x) = \frac{1}{2}kx^2 + 2\lambda L(\cos x - 1)
\]

is a reasonable expression for the potential energy of the system.

(b) Show that equilibrium solutions such that \( V'(x) = 0 \) satisfy the equation

\[ x - \mu \sin x = 0 \]

where \( \mu > 0 \) is a suitable dimensionless parameter. Find and classify the bifurcation point on the branch \( x = 0 \) and give a physical interpretation. Sketch the behavior of the potential \( V(x) \) as \( \mu \) passes through the bifurcation value.

3. (a) A model of a fishery with harvesting is

\[
N_t = \mu N \left(1 - \frac{N}{K}\right) - \frac{HN}{A + N}
\]

where \( N(t) \) is the population of fish at time \( t \) and \( \mu, K, H, A \) are positive parameters. Explain why this is a reasonable model and give a biological interpretation of each of the parameters.

(b) Show that the ODE can be put in dimensionless form

\[
x_t = x(1 - x) - \frac{hx}{a + x}
\]

where \( t \) is a suitably rescaled time and \( a, h \) are dimensionless parameters. Give expressions for \( a, h \) in terms of the original dimensional parameters.

(c) Carry out a bifurcation analysis of the ODE in (b). Discuss the implications of your results for the original fish-harvesting problem.