1. Sketch phase planes of the following $2 \times 2$ linear systems:

(a) \[
\begin{pmatrix}
  x \\
  y
\end{pmatrix}_t = \begin{pmatrix}
  0 & 4 \\
  -9 & 0
\end{pmatrix} \begin{pmatrix}
  x \\
  y
\end{pmatrix};
\]

(b) \[
\begin{pmatrix}
  x \\
  y
\end{pmatrix}_t = \begin{pmatrix}
  0 & 4 \\
  9 & 0
\end{pmatrix} \begin{pmatrix}
  x \\
  y
\end{pmatrix};
\]

(c) \[
\begin{pmatrix}
  x \\
  y
\end{pmatrix}_t = \begin{pmatrix}
  2 & 1 \\
  0 & 2
\end{pmatrix} \begin{pmatrix}
  x \\
  y
\end{pmatrix};
\]

(d) \[
\begin{pmatrix}
  x \\
  y
\end{pmatrix}_t = \begin{pmatrix}
  2 & -1 \\
  -4 & 2
\end{pmatrix} \begin{pmatrix}
  x \\
  y
\end{pmatrix};
\]

(e) \[
\begin{pmatrix}
  x \\
  y
\end{pmatrix}_t = \begin{pmatrix}
  0 & 2 \\
  -5 & 2
\end{pmatrix} \begin{pmatrix}
  x \\
  y
\end{pmatrix};
\]

(f) \[
\begin{pmatrix}
  x \\
  y
\end{pmatrix}_t = \begin{pmatrix}
  0 & 2 \\
  -1 & -3
\end{pmatrix} \begin{pmatrix}
  x \\
  y
\end{pmatrix}.
\]

In each case, classify the equilibrium $(x, y) = (0, 0)$ (as a saddle point, node etc.), determine its stability, and say if it is hyperbolic or non-hyperbolic.

2. Two $n \times n$ linear systems $\ddot{x}_t = A\dot{x}$, $\ddot{y}_t = B\dot{y}$ are said to be differentiably equivalent if there is a diffeomorphism (i.e., a differentiable map with differentiable inverse) $h : \mathbb{R}^n \to \mathbb{R}^n$ such that $\ddot{y}(t) = h(\ddot{x}(t))$ is a solution of $\ddot{y}_t = B\ddot{y}$ if and only if $\ddot{x}(t)$ is a solution of $\ddot{x}_t = A\dot{x}$. Show that if $\ddot{x}_t = A\dot{x}$ and $\ddot{y}_t = B\ddot{y}$ are differentiably equivalent, then $A$ and $B$ have the same eigenvalues. Is differentiable equivalence a useful way to classify the qualitative behavior of linear systems? Explain your answer.

3. Consider the following $2 \times 2$ system of ODEs

$$x_t = x - y, \quad y_t = x + y - 2xy.$$ 

(a) Find the equilibria.
(b) Linearize the system around the equilibria and classify them.
(c) Sketch the phase plane of the system.
(d) Discuss the asymptotic behavior of solutions as $t \to \infty$. Indicate different regions of the phase plane that correspond to different types of asymptotic behavior.