1. Plot the bifurcation diagram and representative phases lines for the scalar ODE
\[ x_t = \mu x + x^3 - x^5. \]
Identify the bifurcation points and classify them. How would the system behave if \( \mu \) is increased quasi-statically from \(-\infty\) to \(+\infty\)? When is the equilibrium \( x = 0 \) linearly stable but unstable to sufficiently large perturbations?

2. Consider the scalar ODE
\[ x_t = \lambda + \mu x - x^2, \]
where \( \lambda, \mu \in \mathbb{R} \) are parameters. Sketch the bifurcation diagram for the equilibria as a function of \( \mu \) for fixed \( \lambda \) in the cases \( \lambda < 0, \lambda = 0, \) and \( \lambda > 0. \) Identify the bifurcation points and classify them in each case.

3. Two rigid rods of length \( L \) are connected by a torsional spring with spring constant \( k \) and are subject to a compressive force of strength \( F. \) Explain why a reasonable model for the potential energy of the system is
\[ V(x) = \frac{1}{2}kx^2 - 2FL(1 - \cos x), \]
where \( x \) is the angle of the rods to the horizontal. If the rod is strongly damped with damping constant \( \beta > 0, \) then the ODE for its motion is \( \beta x_t + V'(x) = 0. \) Show that a nondimensionalized form of the ODE is
\[ x_t + x - \mu \sin x = 0, \quad \mu = \frac{2FL}{k}. \]
Sketch a bifurcation diagram for the ODE and classify the bifurcation that occurs.

4. Consider the system
\[ x_t = 1 - x - \beta xy, \quad y_t = \beta xy - (1 + \gamma)y, \]
where \( \beta, \gamma > 0 \) are positive parameters. Sketch the bifurcation diagram for the equilibria as a function of \( \beta \) and show that a bifurcation occurs at some \( \beta = \beta_*(\gamma) \). What kind of bifurcation is it? Sketch typical phase planes on \( \mathbb{R}^2 \) (using numerical solutions if you prefer) for \( \beta \) close to \( \beta_* \) when \( \beta < \beta_*, \beta = \beta_*, \) and \( \beta > \beta_* \).