Sample Midterm Questions 1  
Math 21B, Winter 2013

1. Suppose that \( f(x), g(x) \) are functions such that
\[
\int_2^0 f(x) \, dx = 2, \quad \int_0^3 g(x) \, dx = 5, \quad \int_2^3 f(x) \, dx = 1.
\]
Evaluate
\[
\int_0^3 \{3f(x) + 2g(x)\} \, dx.
\]

2. Find the following indefinite integrals:
\[
\int \frac{1}{\sqrt{1+x} \left(1+\sqrt{1+x}\right)} \, dx; \quad \int 2\sec^2(5x)\tan(5x) \, dx.
\]

3. Evaluate the definite integral
\[
\int_0^{\pi/2} \left( e^{\cos x} \sin x + e^{\sin x} \cos x \right) \, dx.
\]

4. Suppose
\[
F(x) = \int_0^{x^3} \cos(t^2) \, dt.
\]
Calculate the derivative \( F'(x) \).

5. Find the area enclosed between the graphs
\[
y = x - 1, \quad y = (e - 1) \ln x
\]
for \( 1 \leq x \leq e \).

6. Find the function \( y(x) \) such that
\[
\frac{d^2 y}{dx^2} = e^{-x}, \quad y(0) = 0, \quad \frac{dy}{dx}(0) = 0.
\]
7. (a) Write out a Riemann sum for the integral
\[
\int_{0}^{\pi} \sin(x) \, dx
\]
using a partition of \([0, \pi]\) into 4 equally spaced subintervals and the left endpoints of the subintervals and evaluate it.

(b) Write the integral
\[
\int_{0}^{\pi} \sin(x) \, dx
\]
as a limit of Riemann sums using a partition of \([0, \pi]\) into \(n\) equally spaced subintervals and the left endpoints of the subintervals.

8. Write the limit
\[
\lim_{n \to \infty} \left[ \frac{1}{n^4} \sum_{k=1}^{2n} k^3 \right]
\]
as an integral, and evaluate it.

9. Define a function \(f : [0, 1] \to \mathbb{R}\) by
\[
f(x) = \begin{cases} 
    n & \text{if } x = 1/2^n \text{ for } n = 1, 2, 3, \ldots \\
    0 & \text{otherwise}
\end{cases}
\]
meaning that for \(x = 1/2, 1/4, 1/8, 1/16, \ldots\) we have
\[
f(1/2) = 1, \quad f(1/4) = 2, \quad f(1/8) = 3, \quad f(1/16) = 4, \ldots
\]
and \(f(x) = 0\) otherwise. Is \(f\) Riemann integrable on \([0, 1]\)? If so, what is \(\int_{0}^{1} f(x) \, dx\)?