7.1: 6

Evaluate the integral:

\[ \int \frac{\sec y \tan y}{2 + \sec y} \, dy. \]

This integral can be evaluated using the \( u \)-substitution \( u = 2 + \sec y \). Then \( du = \sec y \tan y \, dy \) and so:

\[ \int \frac{\sec y \tan y}{2 + \sec y} \, dy = \int \frac{1}{u} \, du = \ln u + C = \ln(2 + \sec y) + C. \]

7.1: 26

Evaluate the integral:

\[ \int \frac{1}{1 + e^x} \, dx. \]

There are probably quite a few ways to solve this problem, a fairly simple one is to first simplify (or maybe desimplify is a more accurate term) as follows:

\[ \int \frac{1}{1 + e^x} \, dx = \int \frac{e^{-x}}{e^{-x} + 1} \, dx \]

Now the \( u \)-substitution \( u = 1 + e^{-x} \) gives:

\[ \int \frac{e^{-x}}{e^{-x} + 1} \, dx = -\int \frac{1}{u} \, du = -\ln u + C = -\ln(1 + e^{-x}) + C. \]

If you used a different method and didn’t get the same answer, you still may have gotten an equivalent form. For example, \( x - \ln(1 + e^x) + C \) is equivalent to the answer I gave, though you should be able to figure out why.

7.2: 12

Solve the differential equation:

\[ \frac{dy}{dx} = 3x^2 e^{-y}. \]

We first separate to get that:

\[ e^y \, dy = 3x^2 \, dx. \]

Now we integrate both sides:

\[ \int e^y \, dy = \int 3x^2 \, dx. \]

Luckily both integrals are pretty easy, and we find that:

\[ e^y = x^3 + C. \]

Now we just solve for \( y \):

\[ y = \ln(x^3 + C). \]
The half-life of polonium is 139 days, but your sample will not be useful to you after 95% of the radioactive nuclei present on the day the sample arrives has disintegrated. For about how many days after the sample arrives will you be able to use the polonium?

Although the book gives formulas, I think it’s often best to just do these problems from scratch. Let \( y \) be the amount of polonium you have, and \( t = 0 \) be when the polonium arrives, and we’ll assume that \( y(0) = 100 \). Feel free to choose whatever units you want for this, the number is purely chosen for convenience. I’ll assume we got 100 slugs of polonium. The amount of polonium you have will obey the differential equation:

\[
\frac{dy}{dt} = -ky.
\]

This is solved in the book (although you should be able to solve it yourself by separating variables), giving:

\[
y(t) = y(0)e^{-kt}.
\]

You currently know the half life, which is the time at which we have 50 slugs of polonium left, and we need to find \( k \), the rate constant for this exponential process. So we just solve:

\[
50 \text{ slugs} = 100 \text{ slugs} e^{-k \cdot 139 \text{ days}}
\]

for \( k \), giving:

\[
k = -\frac{\ln(1/2)}{139 \text{ days}} \approx 0.005 \text{ days}^{-1}.
\]

Now we wish to solve for the time at which we have 5 slugs of polonium left, i.e. we solve for \( t \) in the following equation:

\[
5 \text{ slugs} = 100 \text{ slugs} e^{-0.005 \text{ days}^{-1} \cdot t}.
\]

This gives:

\[
t \approx \frac{-1}{0.005 \text{ days}^{-1}} \ln(5/100) = -200 \ln(1/2) \text{ days} \approx 600.
\]

So you’ll have enough polonium for about 600 days.