8.4: 14

Express the integrand as a sum of partial fractions and evaluate the integral.

\[ \int_{1/2}^{1} \frac{y + 4}{y^2 + y} \, dy \]

We first factor:

\[ \frac{y + 4}{y(y + 1)} = \frac{A}{y} + \frac{B}{y + 1} = \frac{A(y + 1) + B y}{y(y + 1)} = \frac{y(A + B) + A}{y(y + 1)}. \]

Now we let \( A \) and \( B \) be given by:

\[ \frac{y + 4}{y(y + 1)} = \frac{A}{y} + \frac{B}{y + 1} = \frac{A(y + 1) + B y}{y(y + 1)} = \frac{y(A + B) + A}{y(y + 1)}. \]

From this we can see that \( A + B = 1 \) and \( A = 4 \). Solving gives that \( B = -3 \). Hence:

\[ \frac{y + 4}{y(y + 1)} = \frac{4}{y} - \frac{3}{y + 1}. \]

Integration is now straightforward:

\[ \int_{1/2}^{1} \frac{y + 4}{y^2 + y} \, dy = \int_{1/2}^{1} \left( \frac{4}{y} - \frac{3}{y + 1} \right) \, dy = (4 \ln y - 3 \ln(y + 1)) \bigg|_{1/2}^{1} = \ln(27/4) \]

8.7: 26

Evaluate the integral:

\[ \int_{0}^{1} (- \ln x) \, dx. \]

This function has a discontinuity at \( x = 0 \), so we need to evaluate it as:

\[ \int_{0}^{1} (- \ln x) \, dx = \lim_{a \searrow 0} \int_{a}^{1} (- \ln x) \, dx. \]

Now we can just directly integrate:

\[ \int_{a}^{1} (- \ln x) \, dx = (x - x \ln x) \bigg|_{a}^{1} = 1 - a + a \ln a. \]

Now we take the limit as \( a \searrow 0 \):

\[ \lim_{a \searrow 0} (1 - a + a \ln a) = 1 + \lim_{a \searrow 0} a \ln a. \]

To evaluate this final limit we must use L'Hôpital's rule:

\[ \lim_{a \searrow 0} a \ln a = \lim_{a \searrow 0} \frac{\ln a}{a} = \lim_{a \searrow 0} -\frac{1/a}{-1/a^2} = \lim_{a \searrow 0} (-a) = 0. \]

Putting this all together gives that:

\[ \int_{0}^{1} (- \ln x) \, dx = 1. \]
11.1: 22

Find a parametrization for the line segment with endpoints \((-1, 3)\) and \((3, -2)\).

There are many ways to do this. Note that \(x\) has to start at \(-1\) and increase by 4, and \(y\) starts at 3 and decreases by 5. Letting \(t\) range between 0 and 4 will make the parametrization of \(x\) easy:

\[x = -1 + t.\]

A parametrization for \(y\) is:

\[y = 3 - \frac{5}{4}t.\]

The factor of \(5/4\) on the \(t\) is necessary so that the \(y\) coordinate changes a bit faster than the \(x\) coordinate. One could check that this parametrization is correct by solving for \(t = x + 1\) and plugging into the second equation; this will give:

\[y = -\frac{5}{4}x + \frac{7}{4},\]

Which is indeed the equation of a line connecting the two given points.

11.2: 26

Find the length of the curve given by \(x = t^3\), \(y = 3t^2/2\), for \(0 \leq t \leq \sqrt{3}\).

The arc length is given by:

\[L = \int_0^{\sqrt{3}} \sqrt{(dx/dt)^2 + (dy/dt)^2} \, dt = \int_0^{\sqrt{3}} \sqrt{(3t^2)^2 + (3t)^2} \, dt = 3 \int_0^{\sqrt{3}} t \sqrt{t^2 + 1}.\]

We can now make the \(u\)-substitution \(u = t^2 + 1\) to evaluate the integral:

\[L = \frac{3}{2} \int_1^4 u^{1/2} \, du = u^{3/2} \bigg|_1^4 = 7.\]

11.3: 40

Replace the polar equation \(r = 4 \tan \theta \sec \theta\) with equivalent cartesian coordinates. Then describe or identify the graph.

We know that \(x = r \cos \theta\), and \(y = r \sin \theta\). So:

\[\frac{y}{x} = \frac{r \sin \theta}{r \cos \theta} = \tan \theta.\]

To find \(\sec \theta\), we note that since \(x = r \cos \theta\), then \(\cos \theta = \frac{x}{r}\). Hence:

\[\sec \theta = \frac{1}{\cos \theta} = \frac{r}{x}.\]

Plugging these results into the equation \(r = 4 \tan \theta \sec \theta\), we find that:

\[r = 4 \frac{y}{x} \cdot \frac{r}{x}.\]

We can then cancel the \(r\)'s and rearrange to find:

\[y = \frac{x^2}{4}.\]

This is a parabola.