1. Determine the interval of convergence (including the endpoints) for the following power series. State explicitly for what values of $x$ the series converges absolutely, converges conditionally, and diverges. In each case, specify the radius of convergence $R$ and the center of the interval of convergence $a$.

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n 2^n}{n} (x - 1)^n$; 
(b) $\sum_{n=0}^{\infty} \frac{1}{3^n + 1} x^{2n}$; 
(c) $\sum_{n=0}^{\infty} \frac{1}{n^2 5^n} (2x + 1)^n$.

2. Let $\vec{u} = 2\vec{i} + 3\vec{k}$, $\vec{v} = \vec{i} + 2\vec{j} + 2\vec{k}$. Compute: (a) $|\vec{u}|$; (b) $|\vec{v}|$; (c) the angle $\theta$ between $\vec{u}$, $\vec{v}$ (you can express it as an inverse trigonometric function); (d) the projection $\text{proj}_\vec{v} \vec{u}$ of $\vec{u}$ in the direction of $\vec{v}$.

3. (a) Find the area of the triangle with vertices $P(-2, 2, 0)$, $Q(0, 1, -1)$ and $R(1, 2, 2)$.
(b) Find a parametric equation for the line in which the planes $3x - 6y - 4z = 15$ and $6x + y - 2z = 5$ intersect.

4. Let $f(x, y) = e^{xy} \ln(y)$. Compute the partial derivatives $f_x$, $f_y$, $f_{xx}$, $f_{xy}$ and $f_{yy}$. (You do NOT need to simplify your answers.)

5. (a) Find the Taylor polynomial $P_4(x)$ of degree 4 centered at $x = 0$ for the function $f(x) = e^{-x^2}$.
(b) Use Taylor’s theorem with remainder to estimate the maximum error $|f(x) - P_4(x)|$ for $0 \leq x \leq 1$.
(c) Use the results of (a) and (b) to obtain an approximate value for the integral

$$\int_0^1 e^{-x^2} dx$$

and estimate the maximum error in your approximate value for the integral.
6. Suppose that the functions $f(x), g(x)$ have the Taylor series expansions at zero, up to second degree terms, given by

$$f(x) = a_0 + a_1 x + a_2 x^2 + \ldots, \quad g(x) = b_0 + b_1 x + b_2 x^2 + \ldots$$

(a) According to Taylor’s theorem, how are $a_0, a_1, a_2$ given in terms of $f$ and its derivatives and $b_0, b_1, b_2$ in terms of $g$ and its derivative?

(b) Find the Taylor series for $h(x) = f(x)g(x)$ at zero, up to second degree terms, by multiplying the Taylor series for $f(x)$ and $g(x)$.

(c) Use the product rule to compute $h'(x), h''(x)$ in terms of the derivatives of $f(x), g(x)$. Show that the use of these expressions in Taylor’s theorem for $h(x)$ gives the same series as the one you found in (b).