# MAT 21D, First Midterm

# October 18, 2019

Name: (Last) (First)

Signature:

Student ID Number: – –

- This room is only for students whose last name starts with a letter A − R. If your last name starts with a letter S − Z, you need to go to Hutchison 115 and take your exam there.
- There needs to be at least one empty seat between any two students.
- Ubiquitous internet access no longer makes bathroom breaks feasible. If you need to use the bathroom, please do so before the start of the exam.
- The exam set consists of 8 pages, including the cover sheet.
- This exam is closed book, no notes, no calculators, no phones or any other wireless devices.
- Use the back sides of the sheets if you need scratch paper.
- Show all your work to obtain full credit.

| Problem | Points | Score |
|---|---|---|
| 1 | 7 | |
| 2 | 7 | |
| 3 | 8 | |
| 4 | 9 | |
| 5 | 10 | |
| 6 | 9 | |
| Total | 50 | |

## Problem 1 [7 Points]

Let $R$ be the region (in the $xy$-plane) bounded by the lines $x = 1$, $x = 2$, $y = 0$, and $y = 2x$.

(a) [1 Point] Sketch $R$.

(b) [6 Points] Evaluate the integral

$$\iint_R e^{1-x^2}\, dA\,.$$

**Problem 2** [7 Points]

Integrate the function

$$f(x, y) = \frac{2x}{y}$$

over the region $R$ in the first quadrant (of the $xy$-plane) bounded by $y = 1$, $y = 2$, $y = 2x$, and $y = x^2$.

## Problem 3 [8 Points]

Consider the region

$$R = \left\{ (x, y) \;\middle|\; 1 \le x^2 + y^2 \le 2,\ x \ge 0,\ y \ge 0 \right\}.$$

Use polar coordinates to evaluate the integral

$$\iint\limits_R \left( \frac{xy}{x^2 + y^2} + 2\cos\bigl(x^2 + y^2\bigr) \right) dA\,.$$

## Problem 4 [9 Points]

Let $D$ be the tetrahedron cut from the first octant (of $xyz$-space) by the plane $2x+2y+z=2$. Evaluate the integral

$$\iiint_D (1-x)\,dV\,.$$

## Problem 5 [10 Points]

Let D be the domain in the first octant (of $xyz$-space) bounded by the coordinate planes and the planes $x = 1$, $y = 1$, and $y + z = 1$.

(a) [1 Point] Sketch $D$.

(b) [3 Points] Find the volume of $D$ by evaluating a suitable triple integral.

(c) [6 Points] Find the average value of the function

$$f(x, y, z) = 2x - y + z$$

over $D$.

## Problem 6 [9 Points]

A solid with constant density $\delta(x, y, z) = \delta > 0$ occupies the domain

$$D = \left\{ (x, y, z) \;\middle|\; x^2 + y^2 \leq 1,\ 0 \leq z \leq 1 - \sqrt{x^2 + y^2} \right\}.$$

(a) [5 Points] Use cylindrical coordinates to find the $z$-component $\overline{z}$ of the centroid of $D$.

(b) [4 Points] Use cylindrical coordinates to find the moment of inertia of the solid about the $z$-axis.