MAT 21D, Second Midterm

November 8, 2019

Name: (Last) (First)

Signature:

Student ID Number: – –

- This room is only for students whose last name starts with a letter A – R. If your last name starts with a letter S – Z, you need to go to Hutchison 115 and take your exam there.
- There needs to be at least one empty seat between any two students.
- Ubiquitous internet access no longer makes bathroom breaks feasible. If you need to use the bathroom, please do so before the start of the exam.
- The exam set consists of 7 pages, including the cover sheet.
- This exam is closed book, no notes, no calculators, no phones or any other wireless devices.
- Use the back sides of the sheets if you need scratch paper.
- Show all your work to obtain full credit.

Problem	Points	Score
1	9	
2	8	
3	8	
4	8	
5	8	
6	9	
Total	50	

Problem 1 [9 Points]

Let C be the curve given by the parametrization

$$r(t) = \left\langle \sqrt{3}\,\sqrt{t},\ \sin\sqrt{t},\ -\cos\sqrt{t}\,\right\rangle, \quad 1 \leq t \leq 9.$$

(a) [4 Points] Find the length of C.

(b) [2 Points] Find the unit tangent vector of C.

(c) [3 Points] Find the curvature of C.

Problem 2 [8 Points]

Consider the domain

$$D = \left\{ (x, y, z) \;\middle|\; 1 \leq x^2 + y^2 + z^2 \leq 4,\; y \geq 0,\; z \geq 0 \right\}$$

in xyz-space. Evaluate the integral

$$\iiint\limits_D \frac{yz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \, dV \, .$$

Problem 3 [8 Points]

Let a, b, $c > 0$ be positive constants. Consider the domain

$$D = \left\{ (x, y, z) \;\middle|\; 1 \leq \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 4,\ z \geq 0 \right\}.$$

(a) [4 Points] Use the transformation $x = au$, $y = bv$, $z = cw$ to rewrite the integral

$$\iiint\limits_D z^2 \, dV$$

as an integral over a domain G in uvw-space. (Do not yet evaluate the integral.)

(b) [4 Points] Use spherical coordinates to evaluate the integral over G from part (a).

Problem 4 [8 Points]

Let C_1 be the line segment joining the points $(0,0,0)$ and $(3,4,0)$, and let C_2 be the line segment joining the points $(3,4,0)$ and $(3,1,4)$. Let $C = C_1 \cup C_2$ be the curve starting in $(0,0,0)$ and ending in $(3,1,4)$, which is formed from C_1 and C_2. Evaluate the integral

$$\int_C \left(\frac{x^2}{3} - \frac{yz}{4} \right) ds\,.$$

Problem 5 [8 Points]

A "thin" wire with variable density is modeled as a curve C with parametrization

$$r(t) = \left\langle \sin t,\ \frac{t^2}{2},\ \cos t \right\rangle, \quad 0 \leq t \leq 2\pi.$$

The density of mass at the point (x, y, z) of C is $\delta(x, y, z) = \dfrac{1}{\sqrt{1 + 2y}}$.

(a) [5 Points] Find the y-component $\overline{y}$ of the center of mass of the wire.

(b) [3 Points] Find the moment of inertia of the wire about the x-axis.

Hint: $\displaystyle\int_0^{2\pi} \cos^2 t \, dt = \pi$

Problem 6 [9 Points]

Let R denote the triangle in the xy-plane with vertices $(0,0)$, $(3,-1)$, and $(-1,2)$.

(a) [4 Points] Find a linear transformation $x = g(u,v)$, $y = h(u,v)$ that maps the three points $(0,0)$, $(1,0)$, and $(0,1)$ in the uv-plane to the three points $(0,0)$, $(3,-1)$, and $(-1,2)$ in the xy-plane.

(b) [5 Points] Use the transformation from (a) to rewrite the integral

$$\iint_R \frac{x+3y}{25}\, dA$$

as an integral over a suitable region G in the uv-plane and evaluate that integral.