This room is only for students whose last name starts with a letter A – R. If your last name starts with a letter S – Z, you need to go to Hutchison 115 and take your exam there.

There needs to be at least one empty seat between any two students.

Ubiquitous internet access no longer makes bathroom breaks feasible. If you need to use the bathroom, please do so before the start of the exam.

The exam set consists of 7 pages, including the cover sheet.

This exam is closed book, no notes, no calculators, no phones or any other wireless devices.

Use the back sides of the sheets if you need scratch paper.

Show all your work to obtain full credit.

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Problem 1

Let $C$ be the curve given by the parametrization

$$r(t) = \langle \sqrt{3}\sqrt{t}, \sin\sqrt{t}, -\cos\sqrt{t} \rangle, \quad 1 \leq t \leq 9.$$

(a) [4 Points] Find the length of $C$.

(b) [2 Points] Find the unit tangent vector of $C$.

(c) [3 Points] Find the curvature of $C$. 
Problem 2

Consider the domain

\[ D = \left\{ (x, y, z) \mid 1 \leq x^2 + y^2 + z^2 \leq 4, \ y \geq 0, \ z \geq 0 \right\} \]

in \( xyz \)-space. Evaluate the integral

\[
\int\int\int_{D} \frac{yz}{(x^2 + y^2 + z^2)^{\frac{3}{2}}} \, dV.
\]
Problem 3

Let $a, b, c > 0$ be positive constants. Consider the domain

$$D = \left\{ (x, y, z) \mid 1 \leq \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} \leq 4, \; z \geq 0 \right\}.$$

(a) [4 Points] Use the transformation $x = au, y = bv, z = cw$ to rewrite the integral

$$\int \int \int_D z^2 \, dV$$

as an integral over a domain $G$ in $uvw$-space. (Do not yet evaluate the integral.)

(b) [4 Points] Use spherical coordinates to evaluate the integral over $G$ from part (a).
Problem 4

Let $C_1$ be the line segment joining the points $(0,0,0)$ and $(3,4,0)$, and let $C_2$ be the line segment joining the points $(3,4,0)$ and $(3,1,4)$. Let $C = C_1 \cup C_2$ be the curve starting in $(0,0,0)$ and ending in $(3,1,4)$, which is formed from $C_1$ and $C_2$. Evaluate the integral

$$\int_C \left( \frac{x^2}{3} - \frac{yz}{4} \right) \, ds.$$
Problem 5

A “thin” wire with variable density is modeled as a curve $C$ with parametrization

$$r(t) = \left\langle \sin t, \frac{t^2}{2}, \cos t \right\rangle, \quad 0 \leq t \leq 2\pi.$$ 

The density of mass at the point $(x, y, z)$ of $C$ is $\delta(x, y, z) = \frac{1}{\sqrt{1 + 2y}}$.

(a) [5 Points] Find the $y$-component $\overline{y}$ of the center of mass of the wire.

(b) [3 Points] Find the moment of inertia of the wire about the $x$-axis.

**Hint:** $\int_{0}^{2\pi} \cos^2 t \, dt = \pi$
Problem 6

Let $R$ denote the triangle in the $xy$-plane with vertices $(0, 0)$, $(3, -1)$, and $(-1, 2)$.

(a) [4 Points] Find a linear transformation $x = g(u, v)$, $y = h(u, v)$ that maps the three points $(0, 0)$, $(1, 0)$, and $(0, 1)$ in the $uv$-plane to the three points $(0, 0)$, $(3, -1)$, and $(-1, 2)$ in the $xy$-plane.

(b) [5 Points] Use the transformation from (a) to rewrite the integral

$$\int \int_{R} \frac{x + 3y}{25} \, dA$$

as an integral over a suitable region $G$ in the $uv$-plane and evaluate that integral.