MAT 21D, Third Midterm

November 22, 2019

Name: (Last) (First)

Signature:

Student ID Number: – –

- Ubiquitous internet access no longer makes bathroom breaks feasible. If you need to use the bathroom, please do so before the start of the exam.
- The exam set consists of 7 pages, including the cover sheet.
- This exam is closed book, no notes, no calculators, no phones or any other wireless devices.
- Use the back sides of the sheets if you need scratch paper.
- Show all your work to obtain full credit.

Problem	Points	Score
1	8	
2	8	
3	9	
4	8	
5	9	
6	8	
Total	50	

Problem 1 [8 Points]

Let C be the curve given by the parametrization

$$r(t) = \left\langle \sin t,\, t^2,\, \cos t \right\rangle, \quad 0 \le t \le 3\pi.$$

Find the work done by the vector field

$$F(x, y, z) = \left\langle x + z,\, y(x^2 + z^2),\, z - x \right\rangle$$

over the curve C in the direction of increasing t.

Problem 2 [8 Points]

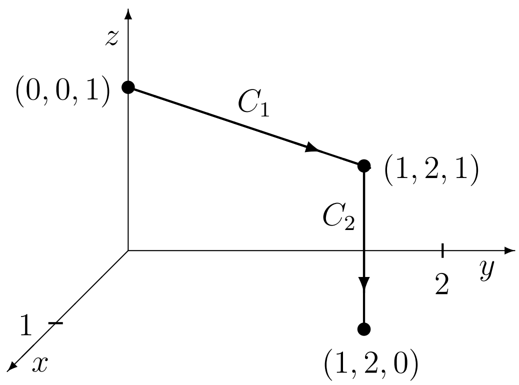
Determine if the following vector fields F in xyz-space are conservative. Give justifications for your answers.

(a) [4 Points] $F(x, y, z) = \left\langle\, 2xz + y^2,\, 2xy - yz^3,\, x^2 + y + z^2 \,\right\rangle$

(b) [4 Points] $F(x, y, z) = \left\langle\, \dfrac{x}{\sqrt{x^2 + z^2 + 1}} + yz,\, xz,\, \dfrac{z}{\sqrt{x^2 + z^2 + 1}} + xy \,\right\rangle$

Problem 3 [9 Points]

Let C_1 and C_2 be the line segments shown in the following figure:



Find the flow of the vector field

$$F(x, y, z) = \left\langle\, 3x - y + z,\ y - xz,\ 2 - xz^2 \,\right\rangle$$

along the curve $C = C_1 \cup C_2$ from $(0, 0, 1)$ to $(1, 2, 0)$.

Problem 4 [8 Points]

Consider the vector field

$$F(x, y, z) = \left\langle 3x^2 z + y^3,\ 3xy^2 - 2yz^2,\ x^3 - 2y^2 z + 2ze^{z^2-1} \right\rangle$$

in xyz-space.

(a) [5 Points] Find a potential function f for F.

(b) [3 Points] Let C be a differentiable curve starting in the point $A = (0, 1, -1)$ and ending in the point $B = (1, 2, 1)$. Find the work W done by F along the curve C. Give a justification for your answer.

Problem 5 [9 Points]

Consider the vector field

$$F(x, y) = \left\langle\, x^3 - 2x - \cos^2 y,\; y^3 - 2y + \sin^2 x \,\right\rangle$$

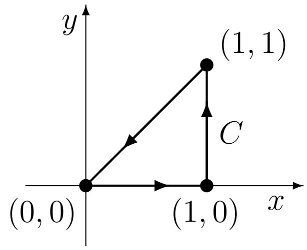
in the xy-plane. Let C be the circle of radius 2 with center at the origin $(0, 0)$, and assume that C is traversed in counterclockwise direction.

(a) [4 Points] Express the flux of F across C as a double integral over a suitable region in the xy-plane. Give a justification for your answer. (Do not yet evaluate the integral.)

(b) [5 Points] Evaluate the double integral from (a) to find the flux of F across C.

Problem 6 [8 Points]

Let C be the closed loop (in the xy-plane) shown in the following figure:



Find the circulation of the vector field $F(x,y) = \langle\, 2x - e^y,\, 4x^2y\,\rangle$ around C.