1. (a) Write down a parametric equation for the ellipse
\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
where \(a, b > 0\) are constants.
(b) Use the parametrization in (a) to obtain an integral for the arclength of the ellipse. Do not attempt to evaluate the integral.

2. Let \(C\) be a curve parametrized by \(-1 < s < 1\) with position vector
\[
r(s) = \frac{1}{3}(1 + s)^{3/2}i + \frac{1}{3}(1 - s)^{3/2}j + \frac{1}{\sqrt{2}}sk.
\]
(a) Show that \(s\) is an arclength parameter for the curve.
(b) Find the unit tangent vector \(T\).
(c) Find the unit normal vector \(N\).
(d) Find the binormal vector \(B = T \times N\).
(e) Find the curvature \(\kappa\).

3. Let
\[
\mathbf{F}(x, y, z) = y^2i + (2xy + z^2)j + (2yz + 3z^2)k.
\]
(a) Compute \(\nabla \times \mathbf{F}\).
(b) Find a function \(f\) such that \(\mathbf{F} = \nabla f\).
(c) Let \(C\) be the oriented line with initial point \((1, -1, 2)\) and terminal point \((-1, 1, -1)\). Evaluate
\[
\int_C \mathbf{F} \cdot d\mathbf{r}.
\]
(d) True or False: “The line integral of \(\mathbf{F}\) around any closed curve is zero.”
4. (a) Let the curve $C$ be the triangle in the $(x, y)$-plane with vertices $(0, 0)$, $(1, 3)$, and $(0, 3)$, oriented counter-clockwise. Use Green’s theorem to write the line integral

$$I = \oint_C \sqrt{x+y} \, dx$$

as an area integral over the interior $R$ of $C$.

(b) Evaluate the area integral in (a). Compare your answer with your answer to Exercise 16, §16.2.

5. Let $R$ be the annulus $a^2 \leq x^2 + y^2 \leq b^2$ in the $(x, y)$-plane, where $0 < a < 1 < b$, and let $\mathbf{F}(x, y) = M(x, y)\mathbf{i} + N(x, y)\mathbf{j}$ be the vector field on $R$ with

$$M(x, y) = -\frac{y}{x^2 + y^2}, \quad N(x, y) = \frac{x}{x^2 + y^2}.$$

(a) Show that

$$\frac{\partial N}{\partial x} = \frac{\partial M}{\partial y}$$

for all $(x, y)$ in $R$.

(b) Let the curve $C$ be circle $x^2 + y^2 = 1$ in $R$, oriented counter-clockwise. Evaluate the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$ 

Is $\mathbf{F}$ a conservative vector field on $R$?

(c) Explain why the results in (a) and (b) are consistent with Green’s theorem.