CALCULUS

Math 21D, Fall, 2019

Sample Questions: Midterm I

1. (a) Evaluate the iterated integral

$$\int_0^1 \int_{x^3}^{\sqrt{x}} y \, dydx.$$

(b) Sketch the region of integration R in the (x, y)-plane determined by the iterated integral in (a) and rewrite the integral with the order of integration reversed. Evaluate it, and show that you get the same result as in (a).

2. (a) Let $0 < a < b$ and $c > 0$ be positive constants. Find the volume under the surface $xyz = c$ over the square $a \leq x \leq b$, $a \leq y \leq b$.

(b) How does the volume over the square $a \leq x \leq b$, $a \leq y \leq b$ compare with the volume over the square $2a \leq x \leq 2b$, $2a \leq y \leq 2b$? Explain.

3. (a) Let R be the circle $x^2 + y^2 \leq \pi$ of radius $\sqrt{\pi}$. Evaluate

$$\int \int_R \sin\left(x^2 + y^2\right) \, dA.$$

(b) What is the average value of $\sin\left(x^2 + y^2\right)$ over R?

4. A wedge of cheese is cut from a cylinder that is 4cm high and 6cm in radius. If the angle of the wedge at the center of the cylinder is $\pi/6$ and the density of the cheese is 2gm/cm^3, compute the mass of the wedge of cheese (include units).

5. Transform the following iterated integral to spherical coordinates and evaluate it:

$$\int_{-3}^{3} \int_{-\sqrt{9-x^2}}^{\sqrt{9-x^2}} \int_0^{\sqrt{9-x^2-y^2}} z\sqrt{x^2 + y^2 + z^2} \, dzdydx.$$

6. Suppose that a function $f(x, y)$ is defined on a closed bounded region R in the plane.

(a) Let $\mathcal{P}$ be a partition of R into rectangles $R_1, R_2, \ldots, R_n$. Define the Riemann sums $S(f, \mathcal{P})$ of f with respect to $\mathcal{P}$.

(b) Define what it means for f to be integrable on R and define $\int\int_R f dA$.

(c) If f is integrable on R and c is a constant, use the definition in (b) to show that

$$\int\int_R cf dA = c \int\int_R f dA.$$