1. Find the general solution of the following ODEs.
   1. $9y'' + 6y' + y = 0$
   2. $y'' + y' - 6y = 0$
   3. $y'' - 14y' + 58y = 0$

2. Find particular solutions of the following ODEs
   1. $y'' + y' + y = 3te^t$
   2. $y'' + 4y = \cos 2t$

3. Find the solutions $\{y_1, y_2\}$ of the ODE
   
   $$y'' - y = 0$$

   which satisfy the following initial conditions
   
   $y_1(0) = 1, \quad y'_1(0) = 0,$
   $y_2(0) = 0, \quad y'_2(0) = 1.$

   Compute the Wronskian of $y_1, y_2$. Is $\{y_1, y_2\}$ a fundamental set of solutions for the ODE? Explain your answer.

4. Consider the sets of functions

   $$S = \{1, t, t^2\}, \quad T = \{t, (1 + t)^2, (1 - t)^2\},$$

   defined for $-\infty < t < \infty$. For each of these sets, say if it is linearly dependent or linearly independent, and prove your answer.

5. Suppose that $p(t), q(t)$ are given coefficient functions, continuous in $-\infty < t < \infty$, such that the functions

   $$y_1(t) = t, \quad y_2(t) = e^{-t^2/2}, \quad y_3(t) = 1$$
satisfy the ODEs
\[
\begin{align*}
y_1'' + p(t)y_1' + q(t)y_1 &= 0, \\
y_2'' + p(t)y_2' + q(t)y_2 &= 0, \\
y_3'' + p(t)y_3' + q(t)y_3 &= \frac{1 - t^2}{1 + t^2}.
\end{align*}
\]
Solve the initial value problem
\[
\begin{align*}
y'' + p(t)y' + q(t)y &= \frac{3 - 3t^2}{1 + t^2}, \\
y(0) &= 0, \\
y'(0) &= 1,
\end{align*}
\]
and explain your answer.

6. Verify that one solution of the ODE
\[
\begin{align*}
ty'' - (t + 1)y' + y &= 0, \\
t &> 0,
\end{align*}
\]
is \(y_1(t) = e^t\). Use the method of reduction of order to find a second linearly independent solution.

7. The displacement \(y(t)\) of an object with mass \(m > 0\) and damping constant \(\gamma > 0\) vibrating on a spring with stiffness \(k > 0\) satisfies the ODE
\[
my'' + \gamma y' + ky = 0.
\]
(a) Find the general solution for \(y(t)\) if \(\gamma^2 > 4km\).

(b) If the solution in (a) is not identically zero, show that the object passes through equilibrium, meaning that \(y(t) = 0\), for at most one time \(t\).