MAT 21C Practice Midterm Exam
Spring, 2015

(The actual exam may be different in length, difficulty, and contents)

Problem 1.

a. The sequence $a_n = \frac{n}{n+1}$ is convergent (to a limit $L$). Find $L$ and determine the value of $N$ such that, for $\varepsilon = 0.1$, $|a_n - L| < \varepsilon$ for $n > N$.

b. Let \{a_k\} be a non-negative sequence, such that $\sum_{k=1}^{\infty} a_k$ is convergent. Determine whether the infinite series $\sum_{k=1}^{\infty} \frac{1}{a_k}$ is convergent.

c. Calculate the value of the convergent series $\sum_{k=2}^{\infty} \frac{2}{k^2 - 1}$

Problem 2.

a. Use the integral test to determine if the following series is convergent $\sum_{k=1}^{\infty} \frac{k}{k^2 + 4}$.

b. Use comparison test to determine if the following series is convergent $\sum_{k=1}^{\infty} \sqrt{\frac{k+4}{k^4 + 4}}$.

c. Test the convergence of the series $\sum_{k=1}^{\infty} \frac{1 + \sqrt{k}}{(k+1)^3 - 1}$

Problem 3. Determine whether each of the following series converges or diverges. Justify your answers.

a. $\sum_{k=1}^{\infty} \frac{(1 + \frac{1}{k})^k}{1}$

b. $\sum_{k=2}^{\infty} \frac{k}{(\ln k)^k}$

Problem 4.

a. Determine the convergence of the alternating series $\sum (-1)^k \frac{k}{(2k+5)^2}$.

b. Determine if the following series are absolutely convergent, conditionally convergent, or divergent.

[i.] $\sum_{k=1}^{\infty} (-1)^k \frac{\arctan k}{k^3 + 2}$

[ii.] $\sum_{k=1}^{\infty} (-1)^{k+1} \sin (\frac{1}{k})$

Problem 5.

a. Consider the power series $f(x) = \sum_{k=10}^{\infty} \frac{(x-1)^k}{\sqrt{k} \cdot 5^k}$.

Determine its radius of convergence, and interval of convergence. Additionally, compute its derivative, $f'(x)$, and determine where the series converges.

b. Write the Maclaurin series (Taylor series at $x = 0$) for $f(x) = x^3e^{-x^2}$. Then, calculate the first four nonzero terms of the Maclaurin series for $\int x^3e^{-x^2} \, dx$. 