METABOLIC RATE OF THE CALIFORNIA CONDOR

In many biological situations, we directly measure one quantity, but we really want to know a function of that measured quantity. For example, we might measure the partial pressure of oxygen in the blood \( p \) but want to know the saturation of hemoglobin, \( S \), which is given by the hemoglobin saturation equation \( S = f(p) = 1/(1 + e^{-ap}) \), where \( a \) is a positive constant that depends on temperature, pH and other properties of the blood.

When using real data to predict the output of a function, measurement error for the independent variable (say, \( x \)) leads to an error in the estimate of the dependent variable (say, \( y \)). This phenomenon, known as error propagation, can be estimated using linear approximation.

a) We can approximate a function near a real number \( x = a \) using the linear approximation of \( f \) around \( a \):

\[
f(x) \approx f(a) + f'(a)(x - a)
\]

i) Use the linear approximation of \( f(x) \) at \( x = a \) to fill in the following lines:

\[
\Delta y = f(a + \Delta x) - f(a)
\approx \quad - f(a)
\]

Note that in this particular context, the term \( f'(a) \) in the final expression is referred to as the sensitivity of \( y \) to \( x \) at \( x = a \).

ii) Conceptually, why do you think we refer this term as sensitivity? What would a larger (or smaller) sensitivity value change about our linear approximation at a particular \( x = a \)?

b) To examine error propagation in more detail, let's consider a model for the metabolic rate of animals. The following curve models how the metabolic rate \( R \) (in kilocalories/day) depends on body mass \( M \) (in kilograms):

\[
R = e^{4.2} M^{0.75}
\]

i) Use this equation to predict the metabolic rate of a California Condor weighing 10kg.

ii) Naturally, not every California Condor weighs exactly 10kg, some weigh more and some weigh less. So, what is the sensitivity of our model to this measurement? Using your equation from part a.i, suitably modified so that body mass \( M \) is the independent variable, metabolic rate \( R \) is the dependent variable and
the functional relationship between the two is \( R = e^{4.2M^{0.75}} \). Select a small error \( \Delta M \) and see how it propagates to an error \( \Delta R \) in our estimate of \( R \).

iii) What are the units of this propagated error, \( \Delta R \)?

**Error vs Percent Error**

c) Note that in parts a and b above, we were discussing \( \Delta R \), which contains units. It’s often more convenient to consider relative error, which does not contain units. That is, suppose that we know that most adult Condors weigh within 10\% of 10kg. Then, we would know that \( \Delta M/M = 0.1 \).

i) Given a 10\% error in the estimate of the condor’s weight, what (approximate) percent error will we have in the metabolic rate? (Hint: if percent error in weight is 100 \( \cdot \) \( \Delta M/M \), what do you think the percent error in metabolic rate is?)