Supplementary Material for Conjugate Gradient Iterative Hard Thresholding: Observed Noise Stability for Compressed Sensing, by J.D. Blanchard, J. Tanner, K. Wei

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I. OUTLINE OF SUPPLEMENTARY MATERIAL

This document contains a representation of the full data generated for [1]. In [1] plots were selected to emphasize the most crucial information contained in the data. For completeness, this document includes all omitted plots. Figs. 1–6 present the 50% recovery phase transition curves for the compressed sensing problem to show the smooth decrease in the recovery region for all algorithms. Figures 7–24, labeled Full data in the list of figures, provide all data for each problem class tested: the 50% recovery phase transition curves for all algorithms, an algorithm selection map identifying the algorithm with minimum average recovery time among all algorithms tested, the minimum average recovery time, and a ratio of the average recovery time for each algorithm compared to the minimum average recovery time among all algorithms tested. For a more detailed view of the recovery performance for all values of \( \rho \) in the phase transition region, the full data also contains semi-log plots of the average computational times for successful recovery for the two values of \( \delta \) which are closest to 0.1 and 0.3.

Consider \( y = Ax + e \) where \( x \in \mathbb{R}^n \) is \( k \)-sparse (i.e. the number of nonzeros in \( x \) is at most \( k \), denoted \( \|x\|_0 \leq k \)), \( A \in \mathbb{R}^{m \times n} \) and \( e \in \mathbb{R}^m \) representing model misfit between representing \( y \) with \( k \) columns of \( A \) and/or additive noise. The compressed sensing recovery question asks one to identify the minimizer

\[
\hat{x} = \arg\min_{z \in \mathbb{R}^n} \|y - Az\|_2 \quad \text{subject to} \quad \|z\|_0 \leq k. \tag{1}
\]

The row-sparse approximation problem extends the compressed sensing problem to consider \( Y = AX + E \) where \( X \in \mathbb{R}^{n \times r} \) is \( k \)-row-sparse (i.e. the number of rows containing nonzero entries in \( X \) is at most \( k \), denoted \( \|X\|_{R0} \leq k \)), \( A \in \mathbb{R}^{m \times n} \) and \( E \in \mathbb{R}^{m \times r} \) representing model misfit between representing \( Y \) with \( k \) columns of \( A \) and/or additive noise. The row-sparse approximation question asks one to identify the \( k \)-row-sparse minimizer

\[
\hat{X} = \arg\min_{Z \in \mathbb{R}^{n \times r}} \|Y - AZ\|_F \quad \text{subject to} \quad \|Z\|_{R0} \leq k. \tag{2}
\]

Question (1) is the special case of (2) with \( r = 1 \).

For the compressed sensing problem (1), the problem classes are defined in [1] and are denoted \((\text{Mat}, B_k)\). The measurement matrix \( A \in \mathbb{R}^{m \times n} \) is drawn from the random matrix ensemble \( \text{Mat} \in \{N, S_T, DCT\} \). \( N \) is the ensemble of dense, Gaussian matrices with entries drawn i.i.d. from \( \mathcal{N}(0, m^{-1}) \). \( S_T \) is the sparse ensemble with seven nonzero values per column drawn with equal probability from \( \{-1/\sqrt{7}, 1/\sqrt{7}\} \) and with locations chosen uniformly. \( DCT \) is the ensemble of randomly subsampled discrete cosine transforms with \( m \) rows of the \( n \times n \) DCT matrix chosen uniformly. The random vector \( x \in \mathbb{R}^n \) is drawn from the sparse binary vector \( B \) with \( k \) locations chosen uniformly and nonzeros of \( \{-1, 1\} \) selected with equal probability. The random vector ensembles \( B \) have the vector \( x \) drawn from \( B \) with the measurements defined by the model \( y = Ax + e \) with \( e \in \mathbb{R}^m \) a random misfit vector drawn uniformly from the sphere of radius \( \epsilon \|Ax\| \).

The matrix ensembles for the row-sparse approximation problem (2) are identical to those from the compressed sensing problem. A problem class \((\text{Mat}, B_k)\) has the measurement matrix \( A \in \mathbb{R}^{m \times n} \) drawn from a random matrix ensemble \( \text{Mat} \in \{N, S_T, DCT\} \). The row-sparse matrix \( X \in \mathbb{R}^{n \times r} \) drawn from the binary row-sparse matrix ensemble has its row-support chosen uniformly with nonzero values \( \{-1, 1\} \) selected with equal probability. For the noise level \( \epsilon \), the measurements are defined by the model \( Y = AX + E \in \mathbb{R}^{n \times r} \) with \( E \) a random misfit matrix; each column \( E_i \) of the misfit matrix \( E \) is drawn uniformly from the sphere of radius \( \epsilon \|Y_i\| \) where \( \{Y_i : i = 1, \ldots, r\} \) are the columns of \( Y = AX \).

For both problems (1) and (2), the sparse ensemble \( B \) is equivalent to the ensemble \( B \) with \( \epsilon = 0 \).

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$\rho = \frac{k}{m}$ 

$\delta = \frac{m}{n}$
Time ratio: CGIHT / fastest algorithm for \((S_7, B_\epsilon)\) \(\epsilon = 0.1, n = 2^{17}\)

(a) 50% phase transition curves for \((S_7, B_\epsilon)\) \(\epsilon = 0.1, n = 2^{17}\). (b) Algorithm selection map. (c) Time (ms) of fastest algorithm for \((S_7, B_\epsilon)\) \(\epsilon = 0.1, n = 2^{17}\).

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(h) Time ratio: NIHT / fastest algorithm for \((S_7, B_\epsilon)\) \(\epsilon = 0.1, n = 2^{17}\).

(i) Time ratio: HTP / fastest algorithm for \((S_7, B_\epsilon)\) \(\epsilon = 0.1, n = 2^{17}\).

(j) Time ratio: CSMPSP / fastest algorithm for \((S_7, B_\epsilon)\) \(\epsilon = 0.1, n = 2^{17}\).

(k) Average recovery time (ms) for \((S_7, B_\epsilon)\) \(\epsilon = 0.1, m = 13108, n = 131072\).

(l) Average recovery time (ms) for \((S_7, B_\epsilon)\) \(\epsilon = 0.1, m = 37666, n = 131072\).

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Time ratio: CGIHT / fastest algorithm for $(N, B, \epsilon)$ $\epsilon = 0.2, n = 2^{10}, r = 10$

Time ratio: CGIHTprojected / fastest algorithm for $(N, B, \epsilon)$ $\epsilon = 0.2, n = 2^{10}, r = 10$

Time ratio: CGITrestarted / fastest algorithm for $(N, B, \epsilon)$ $\epsilon = 0.2, n = 2^{10}, r = 10$

Time ratio: FIHT / fastest algorithm for $(N, B, \epsilon)$ $\epsilon = 0.2, n = 2^{10}, r = 10$

Time ratio: NIHT / fastest algorithm for $(N, B, \epsilon)$ $\epsilon = 0.2, n = 2^{10}, r = 10$

Time ratio: CSMPSP / fastest algorithm for $(N, B, \epsilon)$ $\epsilon = 0.2, n = 2^{10}, r = 10$

Time ratio: HTP / fastest algorithm for $(N, B, \epsilon)$ $\epsilon = 0.2, n = 2^{10}, r = 10$
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