1. Problem 2.2.3 (1 point for each correct answer, with max. 10 points)

Answer:

(a) true (One-sided limits exist and coincide, by looking at the graph. You can also compare to \( f(x) = -|x| \).)

(b) true (Same heuristic as above.)

(c) false (It is true that \( f(0) = 1 \), but this does not mean that \( \lim_{x \to 0} f(x) = 1 \) since the function is not continuous.)

(d) false (Different one-sided limits as the graph immediately shows.)

(e) false (Same reasoning as above.)

(f) true (The function is \( f(x) = |x| \) outside zero and part (b) handles the origin.)

(g) true (Different one-sided limits/jump discontinuity. Only need to reason using the graph again.)

(h) false

(i) true

(j) true

(k) false (Note that \( \lim_{x \to 1^-} f(x) = -1 \), however.)

2. Problems 2.2.20-2.2.21 (10 points each, partial credit for correct use of rules.)

Answer: Using limit rules, we compute

\[
\lim_{z \to 4} \sqrt{z^2 - 10} = \lim_{z \to 4} (z^2 - 10)^{1/2} = (\lim_{z \to 4} z^2 - 10)^{1/2} = (16 - 10)^{1/2} = \sqrt{6},
\]

and

\[
\lim_{h \to 0} \frac{3}{\sqrt{3h + 1} + 1} = \frac{3}{(\lim_{h \to 0} 3h + 1)^{1/2} + 1} = \frac{3}{2}.
\]

(If you use the negative branch of \( \sqrt{\cdot} \), then the answer will not make sense.)
3. Problems 2.2.48-50 (10 points each, partial credit for correct use of rules.)

Answer: We have

$$\lim_{x \to 0} (x^2 - 1)(2 - \cos x) = (0^2 - 1)(2 - \cos 0) = -1 \cdot 1 = -1$$

$$\lim_{x \to -\pi} \sqrt{x + 4 \cos(x + \pi)} = \sqrt{-\pi + 4 \cos(\pi - \pi)} = \sqrt{4 - \pi}$$

$$\lim_{x \to 0} \sqrt{7 + \sec^2 x} = \sqrt{7 + \frac{1}{\cos^2 0}} = \sqrt{8} = 2\sqrt{2}.$$

4. Problem 2.2.51 (use Theorem 2.2.1) (3+4+3 points)

Answer:

(a) Quotient rule
(b) Difference rule + power rule
(c) Sum rule + constant multiple rule

5. Problems 2.2.63-2.2.64 (10 points, partial credit for explanation and correct computations.)

Answer: To get to the form of the statement of the Sandwich Theorem, in Problem 63. we write $g(x) = \sqrt{5 - 2x^2} \leq f(x) \leq \sqrt{5 - x^2} = h(x)$ on the interval $[-1, 1]$. Then we compute

$$\lim_{x \to 0} g(x) = \sqrt{5 - 2 \cdot 0} = \sqrt{5}, \quad \lim_{x \to 0} h(x) = \sqrt{5 - 0} = \sqrt{5}.$$  

By the Sandwich Theorem, we must have $\lim_{x \to 0} f(x) = \sqrt{5}$. In Problem 64., $g(x) = 2-x^2$, $\lim_{x \to 0} g(x) = 2-0^2 = 2$ and $h(x) = 2 \cos x$, $\lim_{x \to 0} h(x) = 2 \cos 0 = 2$. Therefore (by the Sandwich Theorem) $\lim_{x \to 0} f(x) = 2$.

6. Problem 2.3.51 (10 points, no partial credit.)

Definition 1. Let $g(x)$ be a real function. We say that $\lim_{x \to 0} g(x) = k$ if for every $\epsilon > 0$ there exists a corresponding $\delta > 0$ such that whenever $0 < |x| < \delta$, then $|f(x) - k| < \epsilon$. 