MAT 21A Calculus, fall 2018

Practice Final
Material: Thomas’ Calculus 1.1.-4.6, HW 1-7, lectures

NB: There are more problems in this mock exam than will be on the real one.

1. Compute the following limits.
   (a) \( \lim_{x \to 3} e^{1/x} \)
   (b) \( \lim_{x \to 3^+} \ln(x - 3) \)
   (c) \( \lim_{x \to 3} \frac{\ln(x-2)}{x-3} \)
   (d) \( \lim_{x \to \infty} \frac{8x^5 - 7x^3 + 9}{(3x^2 - 1)(2x^3 - 3)} \)
   (e) \( \lim_{x \to \infty} \frac{e^x}{x^3} \)
   (f) \( \lim_{x \to 0} \frac{\arctan(x-x)}{x^3} \)

2. Compute the derivatives of the following functions.
   (a) \( y = x \ln x - x \)
   (b) \( y = e^{3x^2} \)
   (c) \( y = (x - 1)^5 \cos x \)
   (d) \( y = \sin(\ln x) \)
   (e) \( y = \frac{e^{3x^2 + 2}}{\cos x + 2} \)
   (f) \( y = \sin(e^x - x^2 - 1) \)
   (g) \( y = \frac{\arctan(4x)}{x^2 + 1} \)

3. Find the absolute minima and maxima of the functions on the given domain.
   (a) \( f(x) = x^2 e^{-x} \) on \([0, 1]\)
   (b) \( f(x) = 2x^3 - 3x^2 \) on \([-1, 2]\)
   (c) \( f(x) = \sin^2 x \) on \([0, \pi]\)
   (d) \( f(x) = \frac{x}{1 + x^2} \) on \([-2, 2]\).

4. Find the equation of the tangent line to the curve at the given point:
   (a) \( y = \ln x \) at \( x = 5 \).
   (b) \( \ln(x^2 + y - 1) + xy + y^4 = 2 \) at \( (x, y) = (1, 1) \).
5. For the given function $f(x)$:

- Find the domain of $f(x)$.
- Find the derivative of $f(x)$ and determine the open intervals where $f(x)$ is increasing/decreasing using a sign chart. Determine also the critical points.
- Find the second derivative of $f(x)$ and determine the open intervals where $f(x)$ is concave up/down. Determine the inflection points.
- * Good practice but not on the final* Graph the function using the information above.

(a) $f(x) = 2x^3 - 3x^2 + 1$
(b) $f(x) = xe^{-x}$
(c) $f(x) = \ln(x^2 + 1)$
(d) $f(x) = \frac{2x}{x+1}$

6. Consider the function

$$
\begin{cases}
  x + 1, & \text{if } x < -1, \\
  x^2 + ax + b, & \text{if } x \geq -1.
\end{cases}
$$

(a) For which values of the parameters $a, b$ is this function continuous?
(b) For which parameter values is the function differentiable everywhere?

7. Consider the curve given by the equation $x^{2/3} + y^{2/3} = 1$. Find $y'$ using implicit differentiation and find the equation of the tangent line to the curve at $(x, y) = (0, 1)$. Sketch the curve.

8. You are designing a rectangular poster to contain 50in$^2$ of printing with 4in margins at the top and bottom and 2in margins at each side. What overall dimensions will minimize the amount of paper used?

9. An open rectangular box with square base is to be made from wood. There is a total of 1m$^2$ wood available. What dimensions of the box will result in a box with the largest possible volume?

10. A TV set costs 100 dollars. If its price is lowered by $a\%$, the sales will increase by $2a\%$. Find the discount percentage $a$ resulting in maximal profit.

11. The surface area of a cube is increasing at the rate of 6ft$^2$/min. Determine the rate at which the volume of the cube is changing, when the edge of the cube is 2ft long.

12. Among all tangent lines to the graph of $y = \frac{1}{1+x^2}$, find the equation of the tangent line with minimal slope.