ESP
Kouba
Worksheet 10

1. Differentiate.

a. $y = \left(\dfrac{3x - 1}{\tan 5x}\right)^7$

b. $f(x) = \tan\left\{\dfrac{\sin(3x+7)}{\cos(7x-3)}\right\}$

c. $g(x) = \sin(\sin(\sin(\sin(1/x))))$

d. $y = \csc^3 \sqrt{1-x^2}$

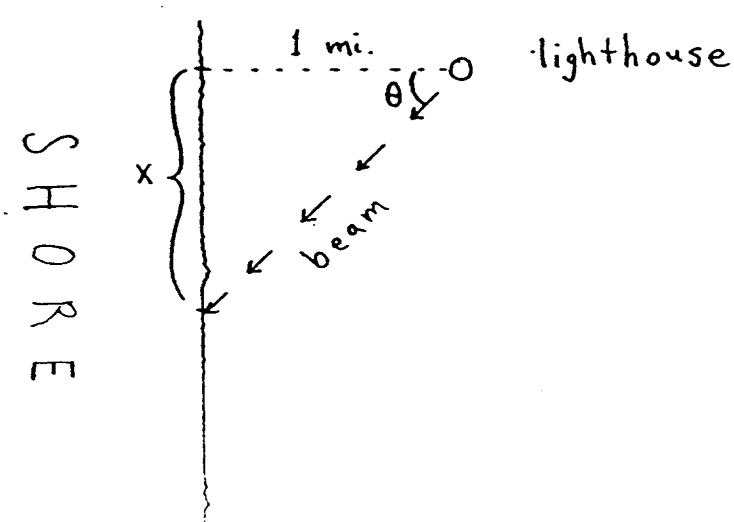
2. Consider the function $f(x) = \dfrac{x}{x^2+1}$.

a. Sketch its graph.
b. What points (x, y) on the graph of f determine horizontal tangent lines ?
c. What points (x, y) on the graph of f determine tangent lines with slope 1/2 ?
d. What points (x, y) on the graph of f determine tangent lines with slope - 1/10 ?
e. What is the smallest possible value of $f'(x)$?

3. A lighthouse sits one mile offshore with a light beam turning counter-clockwise at the rate of ten revolutions per minute. See diagram.

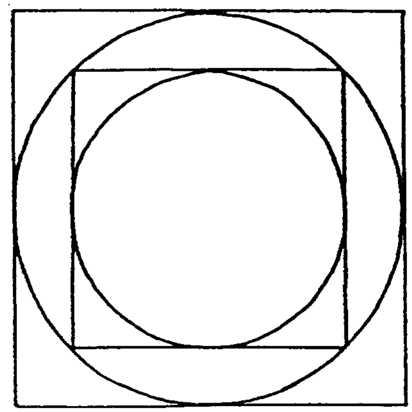
a. Write distance x as a function of θ .
b. Assume that both x and θ are functions of time t .

i. Determine dθ/dt .
ii. Determine dx/dt .



4. Gooh has a density of 200 grams per liter and sludge has a density of 250 grams per liter. What combination of gooh and sludge will result in 10 liters of mixture (slooh ?!) having a density of 238 grams per liter ?

5. Consider the diagram of nested circles and squares. The larger square has side length 2. Determine the area of the smaller circle.



6. Find all values of c guaranteed by the Mean Value Theorem for the following functions and intervals.

a. $f(x) = x^2 - x$ on $[0, 2]$

b. $f(x) = x^{1/3} + 1$ on $[-2, 0]$

c. $f(x) = x + 1/x$ on $[1/2, 3]$

d. $f(x) = 5 \sin 3x$ on $[0, \pi]$

e. $f(x) = \cos(2x + 1)$ on $[0, \pi/2]$

7. Let $d(t) = 10t^2 + 3\sqrt{2t}$ be the distance (miles) a bicyclist has traveled after t hours. Prove that, at least once during the first two hours of travel, the bicyclist reaches a speed of 23 miles per hour.

8. Let f and g be continuous functions with $g(x) \neq 0$ on the closed interval $[a, b]$ and with f and g differentiable on the open interval (a, b). Assume that

$$f(a) = g(a) \text{ and } f(b) = g(b).$$

Show that there is some number c in (a, b) satisfying

$$g(c)\, f'(c) = f(c)\, g'(c).$$

9. Determine a function f whose derivative is given by f ' .

a. $f'(x) = 3x^2 - 1$

b. $f'(x) = \pi + \sqrt{x}$

c. $f'(x) = 5 \cos(5x - 7)$

d. $f'(x) = x \sec^2 x + \tan x$

e. $f'(x) = \dfrac{x \cos x - \sin x}{x^2}$

10. Determine a function y satisfying $y' = \dfrac{1}{2y}$.

11. Prove that the equation $x^5 - \sqrt{x} = x^{1/3} + 1$ has at least one solution c.

12. Sketch the graph of $g(x) = \begin{cases} \sin x & \text{for } x \geq \pi \\ \pi - x & \text{for } x < \pi \end{cases}$ and prove that g is differentiable at $x = \pi$.

13. Sketch the graph of $f(x) = x^{1/5}$ and prove that f is not differentiable at $x = 0$.

14. Tarzan can clean the bamboo hut in five hours. Jane can clean the hut in three hours. How long does it take the two of them working together to clean the hut ?

15. a. Use your calculator to evaluate the first seven terms of the following sequence :

$$0.8\ ,\ 0.8^{0.8}\ ,\ 0.8^{0.8^{0.8}}\ ,\ 0.8^{0.8^{0.8^{0.8}}}\ ,\ \ldots$$

Estimate what you think the limit of this sequence is.

b. Let c be a constant so that the limit of the sequence

$$c\ ,\ c^{c}\ ,\ c^{c^{c}}\ ,\ c^{c^{c^{c}}}\ ,\ \cdots \quad \text{is } 2\,.$$

Determine the value of c .