

ESP
Kouba
Worksheet 11

1. Determine the values of x for which the following functions are increasing, decreasing, concave up, and concave down. Indicate relative maximum, relative minimum, and inflection points. Neatly sketch the graph.

a. $f(x) = x^2(x - 3)$

b. $f(x) = \frac{x}{x^2 + 1}$

c. $f(x) = 2x - 1/x$

d. $f(x) = x - \sqrt{1 - x^2}$

2. Determine a function f whose derivative is given by f' .

a. $f'(x) = \pi^2 - e^2$

b. $f'(x) = 1 + x + x^2$

c. $f'(x) = \sec^2 7x$

d. $f'(x) = 2 \sin x \cos x$

e. $f'(x) = 3(f(x))^2$

3. Let $f(x) = \frac{x^3}{x+1}$. For what values of x is $f'(x) = 0$?

4. Solution A is 35% alcohol (by volume) and solution B is 60% alcohol (by volume). How much of each solution should be combined in order to have two liters of solution which is 40% alcohol (by volume)?

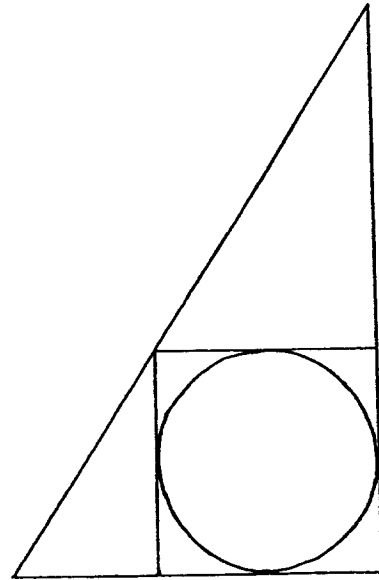
5. Let $f(x) = g(h(x))$. Assume that $h(1) = 2$, $g(2) = -3$, $h'(1) = -1$, $g'(2) = 1/2$, $h''(1) = 3$, and $g''(2) = -2$. Determine values for each of the following.

a. $f(1)$

b. $f'(1)$

c. $f''(1)$

6. A circle is inscribed in the square, which is inscribed in the right triangle, which has height 2 and base 1. Determine the circumference of the circle. See diagram.



7. Prove rigorously that the following statements about limits are true.

a. $\lim_{x \rightarrow 2} (3x-1) = 5$

b. $\lim_{x \rightarrow -\frac{1}{2}} (4+2x) = 3$

c. $\lim_{x \rightarrow 1} (2x^2-3) = -1$

d. $\lim_{x \rightarrow -2} (13-3x^2) = 1$

e. $\lim_{x \rightarrow 3} \frac{2}{x-1} = 1$

f. $\lim_{x \rightarrow -\frac{1}{2}} \frac{x-1}{x+1} = -3$

g. $\lim_{x \rightarrow 4} (7-3\sqrt{x}) = 1$

h. $\lim_{x \rightarrow \frac{1}{2}} \frac{x^2}{2x+3} = \frac{1}{16}$