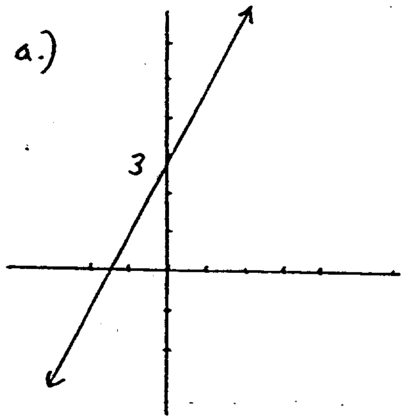
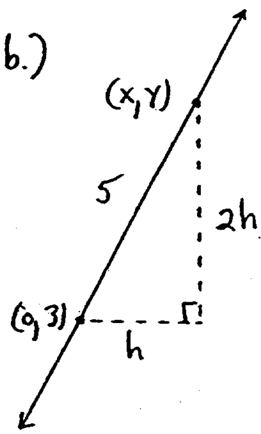
ESP
Kouba
Worksheet 12 Solutions

1.) $Y = 2X \cdot \cos 3X \Rightarrow Y' = 2X \cdot {}^{-}\sin 3X \cdot 3 + 2 \cdot \cos 3X$ and $X = \frac{\pi}{2}$, $Y = 2 \cdot \frac{\pi}{2} \cdot \cos \frac{3\pi}{2} = \pi \cdot 0 = 0$, $Y' = {}^{-}6 \cdot \frac{\pi}{2} \cdot \sin \frac{3\pi}{2} + 2 \cdot \cos \frac{3\pi}{2} = 3\pi \Rightarrow$ tangent line is $Y - 0 = 3\pi\left(X - \frac{\pi}{2}\right)$.

2.) a.)



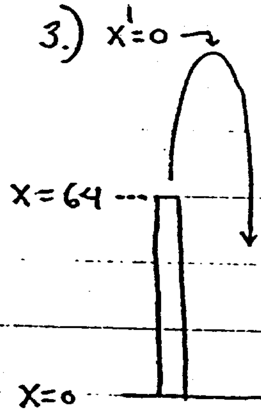
b.)



$h^2 + (2h)^2 = 5^2 \rightarrow$
$5h^2 = 25 \rightarrow$
$h = \sqrt{5}$ so
points (x, y) are

$(0 \pm \sqrt{5},\ 3 \pm 2\sqrt{5})$ or $(\sqrt{5},\ 3 + 2\sqrt{5})$ and $(-\sqrt{5},\ 3 - 2\sqrt{5})$.

3.)



acc.: $x''(t) = -32$ ft./sec².
vel.: $x'(t) = -32t + c$ and $x'(0) = 48$ ft./sec.
$\rightarrow 48 = -32(0) + c \rightarrow c = 48$ so
$\boxed{x'(t) = -32t + 48}$
pos.: $x(t) = -16t^2 + 48t + c$ and $x(0) = 64$ ft.
$\rightarrow 64 = -16(0) + 48(0) + c \rightarrow c = 64$ so
$\boxed{x(t) = -16t^2 + 48t + 64}$;

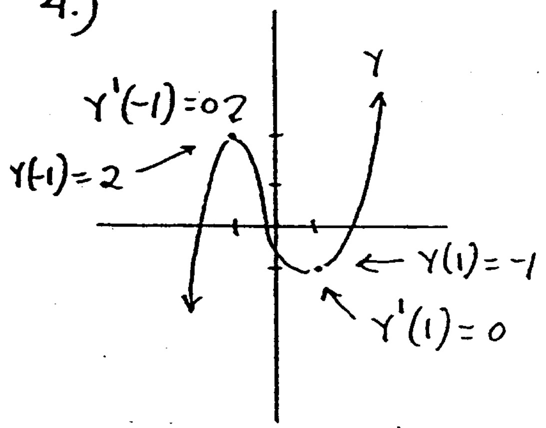
a.) $x'(t) = 0 \rightarrow -32t + 48 = 0 \rightarrow t = \frac{48}{32} = \frac{3}{2}$ sec.

b.) $x\left(\frac{3}{2}\right) = -16\left(\frac{3}{2}\right)^2 + 48\left(\frac{3}{2}\right) + 64 = 100$ ft.

c.) $x(t) = 0 \rightarrow -16t^2 + 48t + 64 = 0 \rightarrow$
$-16(t^2 - 3t - 4) = -16(t - 4)(t + 1) = 0 \rightarrow t = 4$ sec.

d.) $x'(4) = -32(4) + 48 = -80$ ft./sec.

4.)



$Y = Ax^3 + Bx^2 + Cx + D$

$Y' = 3Ax^2 + 2Bx + C$

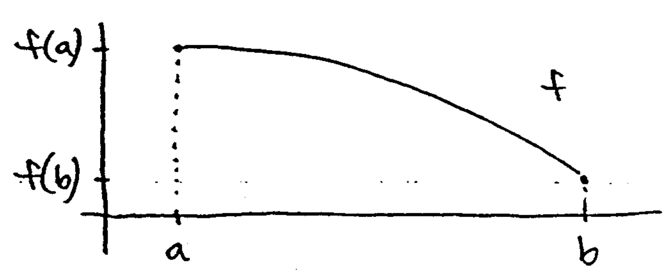
$$\left.\begin{array}{ll} Y(1)=-1: & A+B+C+D=-1 \\ Y(-1)=2: & -A+B-C+D=2 \\ Y'(1)=0: & 3A+2B+C=0 \\ Y'(-1)=0: & 3A-2B+C=0 \end{array}\right\}$$

$$\left.\begin{array}{l} 2A+2C=-3 \\ 6A+2C=0 \end{array}\right\} \quad 4A = 3 \rightarrow \quad A = 3/4 \text{ and } C = -\frac{9}{4} \rightarrow$$

$$\left.\begin{array}{l} \frac{3}{4} + B - \frac{9}{4} + D = -1 \\ -\frac{3}{4} + B + \frac{9}{4} + D = 2 \\ \frac{9}{4} + 2B - \frac{9}{4} = 0 \end{array}\right\} \quad \begin{array}{l} 2B + 2D = 1 \\ B = 0 \text{ so } D = \frac{1}{2} \rightarrow \end{array}$$

$$Y = \frac{3}{4}x^3 - \frac{9}{4}x + \frac{1}{2}$$

5.) a.) $f'(x) < 0$ for $a < x < b$ so f is decreasing



b.) By MVT there is c in (a,b) satisfying

$$f'(c) = \frac{f(b) - f(a)}{b-a} < 0 \quad \text{(by part a.))} \rightarrow$$

$$f(b) - f(a) < (b-a)\cdot 0 \rightarrow f(b) - f(a) < 0 \rightarrow f(b) < f(a).$$

6.) $Y' = 5(1+\sin 8x)^4 \cdot \cos 8x \cdot 8 = 40 \cdot \cos 8x \cdot (1+\sin 8x)^4$

at $x = \frac{\pi}{24}$ slope of tangent line is

$Y' = 40 \cdot \cos\frac{\pi}{3} \cdot \left(1+\sin\frac{\pi}{3}\right)^4 = 40 \cdot \frac{1}{2}\left(1+\frac{\sqrt{3}}{2}\right)^4 = 20\left(1+\frac{\sqrt{3}}{2}\right)^4$

so slope of normal line is $\dfrac{-1}{20\left(1+\frac{\sqrt{3}}{2}\right)^4}$.

7.) $f'(x) = 2 - \sec^2 x = 0 \rightarrow \sec^2 x = 2 \rightarrow \sec x = \pm\sqrt{2} \rightarrow$

$\cos x = \frac{\pm 1}{\sqrt{2}} \rightarrow x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}, \ldots$

8.) f is continuous on $[1,2]$ and differentiable on $(1,2)$ so by MVT there is some c in $(1,2)$ (and hence in $[0,3]$) satisfying

$$f'(c) = \frac{f(2)-f(1)}{2-1} = f(2)-f(1).$$

9.) a.) $N(0) = 370$ elk, $N(10) = 78.66 \approx 78$ elk

b.) $\dfrac{N(5)-N(1)}{5-1} = \dfrac{192.11-356.79}{4} = -41.17 \approx -41$ elk/yr.

c.) $N'(t) = -9990(t^2+27)^{-2} \cdot 2t = \dfrac{-19980t}{(t^2+27)^2}$ so

$N'(1) = -25.48$ elk/yr. and

$N'(5) = -36.95$ elk/yr.

d.) The number of elk changes most rapidly at the inflection point of the graph of N:

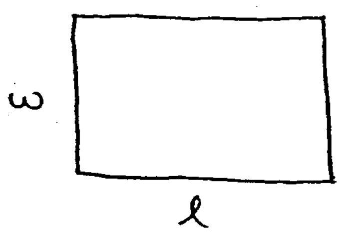
$$N''(t) = \frac{(t^2+27)^2\cdot(-19980)-(-19980t)\cdot 2(t^2+27)\cdot 2t}{(t^2+27)^4}$$

$$= \frac{-19980\,(t^2+27)\left[(t^2+27)-4t^2\right]}{(t^2+27)^4} = \frac{-19980\,(27-3t^2)}{(t^2+27)^3}$$

$$= \frac{-59940\,(9-t^2)}{(t^2+27)^3} = 0 \rightarrow t = 3 \text{ years}$$

(and $N'(3) = -46.25$ elk/yr.)

10.)



$2w + 2l = 400 \rightarrow w = 200 - l$

and maximize area)

$A = wl = (200-l)\,l$

$= 200l - l^2 \rightarrow$

$A' = 200 - 2l = 0 \rightarrow l = 100$

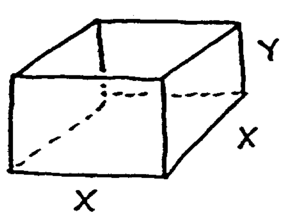
$+ \quad 0 \quad -$ $\quad A'$

$l = 100$ ft.

$w = 100$ ft.

max. $A = 10{,}000$ ft.^{2}

11.)



$X^2 + 4XY = 12 \rightarrow Y = \dfrac{12-X^2}{4X}$

and maximize volume

$V = X^2Y = X^2\cdot\dfrac{12-X^2}{4X} = \frac{1}{4}X(12-X^2) \rightarrow$

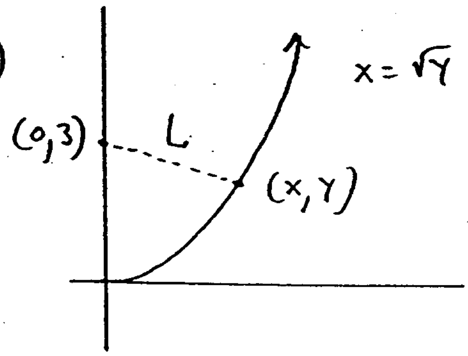
$V' = \frac{1}{4}X(-2X) + \frac{1}{4}(12-X^2) = 3 - \frac{3}{4}X^2 = 0 \rightarrow X = 2$

$+ \quad 0 \quad -$ $\quad V'$

$X = 2$ ft.

$Y = 1$ ft.

max. $V = 4$ ft.^{3}

12.) 

Minimize distance

$$L = \sqrt{(Y-3)^2 + (x-0)^2}$$
$$= \sqrt{(Y-3)^2 + (\sqrt{Y})^2}$$
$$= \sqrt{(Y-3)^2 + Y} \rightarrow$$

$$L' = \frac{1}{2}\left((Y-3)^2+Y\right)^{-\frac{1}{2}} \cdot \{2(Y-3)+1\} = \frac{2Y-5}{2\sqrt{(Y-3)^2+Y}} = 0 \rightarrow$$

$2Y - 5 = 0 \rightarrow$

$Y = 5/2, \quad X = \sqrt{\frac{5}{2}}$

$-$ 0 $+$ L'

$Y = 5/2$

min. $L = \sqrt{\frac{1}{4} + \frac{5}{2}} = \frac{\sqrt{11}}{2}$

13.)

\$ per room	# of rooms	total \$
\$50	100	\$5000
\$55	97	\$5335
\$60	94	\$5640
\$100	70	\$7000
\$125	55	\$6875

Let X be the # of \$5 increase in room cost so that daily revenue will be

R = (charge per room)(# of rooms)

$R = (50+5x)(100-3x) \rightarrow$

$R' = (50+5x)(-3) + 5(100-3x) = 350 - 30X = 0 \rightarrow$

$X = 11\frac{2}{3}$

$+$ 0 $-$ R'

$X = 11\frac{2}{3}$

room charge : \$108.33

# of rooms : 65

max. revenue : \$7041.45

14.) a.) i.) $x^2 - x - 6 = (x-3)(x+2) = 0 \rightarrow x = 3,\ x = -2$

ii.) $2x^2 - 3x + 1 = (2x-1)(x-1) = 2(x-\tfrac{1}{2})(x-1) = 0 \rightarrow$

$x = \tfrac{1}{2},\ x = 1$.

b.) If f is a quadratic with roots r and s then

$$f(x) = c(x-r)(x-s) \qquad (c \text{ some constant})$$

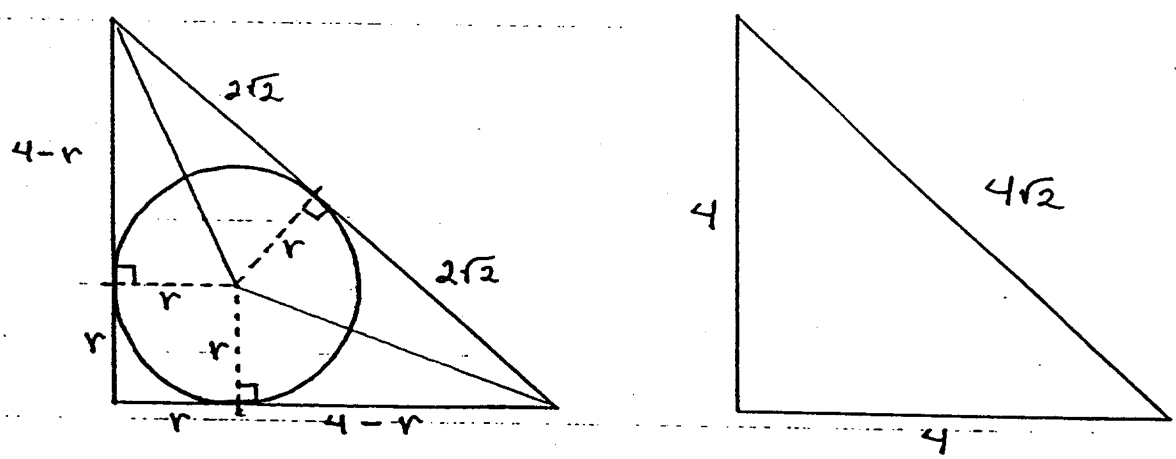
$$\rightarrow f'(x) = c(x-r)(1) + c(1)(x-s) = c(x-r) + c(x-s)$$

$$\rightarrow f'(r) + f'(s) = \{c(\overset{0}{\cancel{r-r}}) + c(r-s)\} + \{c(s-r) + c(\overset{0}{\cancel{s-s}})\}$$

$$= c(r-s) + c(s-r)$$

$$= \cancel{cr} - \cancel{cs} + \cancel{cs} - \cancel{cr} = 0.$$

15.)



The large triangle is composed of a small square and 4 small right triangles, so area of large triangle is

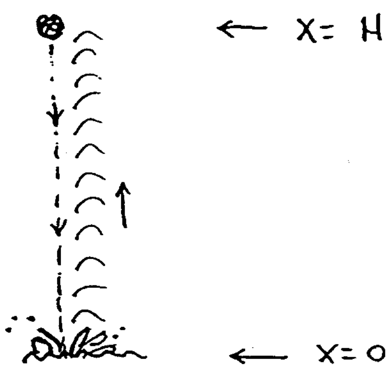
$$\tfrac{1}{2}(4)(4) = \text{area square} + \text{areas of rt. } \Delta\text{'s}$$

$$= r^2 + 2\{\tfrac{1}{2}(4-r)r + \tfrac{1}{2}(2\sqrt{2})r\} \rightarrow$$

$$8 = \cancel{r^2} + 4r - \cancel{r^2} + 2\sqrt{2}\, r \;\rightarrow\; 8 = (4 + 2\sqrt{2})\, r \;\rightarrow$$

$$r = \frac{4}{2+\sqrt{2}} \approx 1.17 .$$

16.)



H : ht. of bridge ,
T : time hit H_2O ,
10−T : time for sound to travel ;
Assume speed of sound is
1115 ft./sec.

$$x''(t) = -32$$
$$x'(t) = -32t$$
$$x(t) = -16t^2 + H$$

then

$$x(T) = 0 \;\rightarrow\; \boxed{-16T^2 + H = 0}$$

and

sound : $\boxed{H = (1115)(10-T)}$ $\rightarrow$

$$16T^2 + 1115T - 11150 = 0 \;\rightarrow$$

$$T = \frac{-1115 + \sqrt{1956825}}{32} = 8.87 \text{ seconds}$$

and $H = 1259$ feet