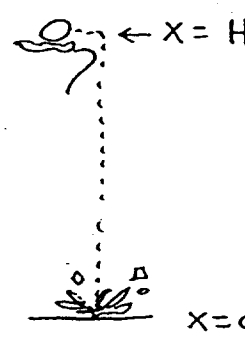


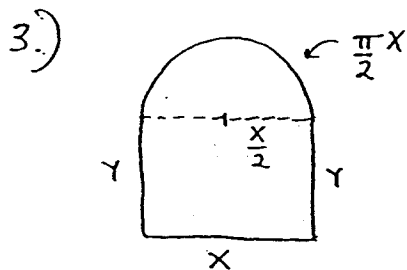
1.) $f'(x) = 2 \sec 2x \cdot \tan 2x = 0 \rightarrow \sec 2x = 0$ or $\tan 2x = 0$
 $\rightarrow 2x = 0, \pm\pi, \pm 2\pi, \pm 3\pi, \dots \rightarrow x = 0, \pm\frac{\pi}{2}, \pm\pi, \pm\frac{3\pi}{2}, \dots$

2.)  $x''(t) = -32$ H: height
 $x'(t) = -32t$ T: falling time
 $x(t) = -16t^2 + H$

60 mph = 88 ft./sec.

$x(T) = 0 \rightarrow \boxed{0 = -16T^2 + H}$ and
 $x'(T) = -88 \rightarrow \boxed{-32T = -88} \rightarrow$

$T = 2.75 \text{ sec} \rightarrow H = 121 \text{ feet}$



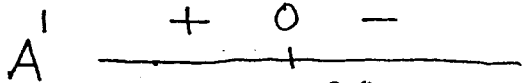
$2y + x + \frac{\pi}{2}x = 10 \rightarrow$
 $y = 5 - \frac{1}{2}x - \frac{\pi}{4}x$ and
 maximize total area

$A = xy + \frac{1}{2}\pi\left(\frac{x}{2}\right)^2 \rightarrow$

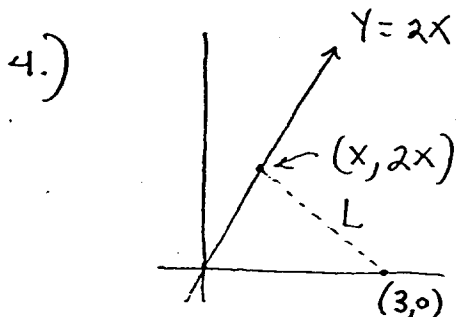
$A = x\left(5 - \frac{1}{2}x - \frac{\pi}{4}x\right) + \frac{\pi}{8}x^2 = 5x - \frac{1}{2}x^2 - \frac{\pi}{4}x^2 + \frac{\pi}{8}x^2 \rightarrow$

$A = 5x - \frac{1}{2}x^2 - \frac{\pi}{8}x^2 \rightarrow$

$A' = 5 - x - \frac{\pi}{4}x = 0 \rightarrow x = \frac{5}{1 + \frac{\pi}{4}} = \frac{20}{4 + \pi}$



$x = \frac{20}{4 + \pi} \approx 2.8 \text{ ft}, y = \frac{10}{4 + \pi} \approx 1.4 \text{ ft}, \text{ max. } A \approx 7 \text{ ft.}^2$



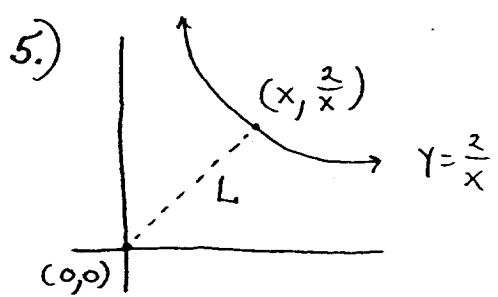
Minimize distance

$L = \sqrt{(2x-0)^2 + (x-3)^2}$
 $= \sqrt{4x^2 + (x-3)^2} \rightarrow$

$L' = \frac{1}{2}(4x^2 + (x-3)^2)^{-\frac{1}{2}} \cdot \{8x + 2(x-3)\} = \frac{10x-6}{2\sqrt{4x^2 + (x-3)^2}} = 0 \rightarrow$

$10x - 6 = 0 \rightarrow x = \frac{3}{5}$

$$L' \quad \frac{- \quad 0 \quad +}{x = 3/5, \quad y = 6/5, \quad \text{min. } L = \sqrt{\frac{180}{25}} = \frac{6\sqrt{5}}{5}$$



minimize distance

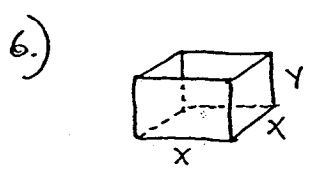
$$L = \sqrt{\left(\frac{2}{x} - 0\right)^2 + (x - 0)^2}$$

$$= \sqrt{\frac{4}{x^2} + x^2} \rightarrow$$

$$L' = \frac{1}{2} \left(\frac{4}{x^2} + x^2\right)^{-1/2} \cdot \left\{ \frac{-8}{x^3} + 2x \right\} = \frac{2x^4 - 8}{2x^3 \sqrt{\frac{4}{x^2} + x^2}} = 0 \rightarrow$$

$$2x^4 - 8 = 0 \rightarrow x^4 = 4 \rightarrow x = 4^{1/4}$$

$$L' \quad \frac{0}{x = 4^{1/4} \approx 1.4, \quad y = \frac{2}{4^{1/4}} \approx 1.4, \quad \text{min. } L = 2$$

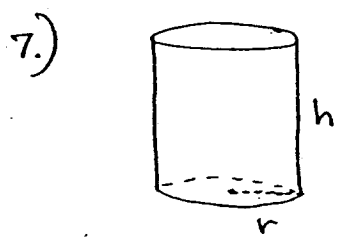


$x^2 y = 4 \rightarrow y = \frac{4}{x^2}$
and minimize surface area

$$A = x^2 + 4xy = x^2 + 4x \cdot \frac{4}{x^2} = x^2 + \frac{16}{x} \rightarrow$$

$$A' = 2x - \frac{16}{x^2} = \frac{2x^3 - 16}{x^2} = 0 \rightarrow 2x^3 - 16 = 0 \rightarrow x = 2$$

$$A' \quad \frac{- \quad 0 \quad +}{x = 2 \text{ ft.}, \quad y = 1 \text{ ft.}, \quad \text{min. } A = 12 \text{ ft.}^2$$



$$\pi r^2 + 2\pi r h = 3\pi \rightarrow$$

$$h = \frac{3\pi - \pi r^2}{2\pi r} = \frac{3 - r^2}{2r}$$

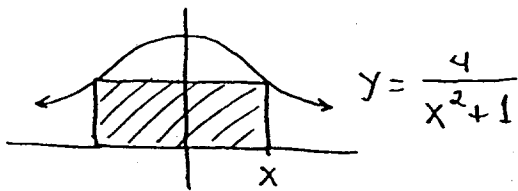
and maximize volume

$$V = \pi r^2 h = \pi r^2 \cdot \frac{3 - r^2}{2r} = \frac{3}{2} \pi r - \frac{\pi}{2} r^3 \rightarrow$$

$$V' = \frac{3}{2} \pi - \frac{3}{2} \pi r^2 = \frac{3}{2} \pi (1 - r^2) = 0 \rightarrow r = 1$$

$$V' \quad \begin{array}{c} + \quad 0 \quad - \\ \hline r=1 \text{ m.}, \quad h=1 \text{ m.}, \quad \text{max. } V = \pi \text{ m.}^3 \end{array}$$

8.)



Maximize area of rectangle

$$A = 2x \cdot y = 2x \cdot \frac{4}{x^2+1} = \frac{8x}{x^2+1} \rightarrow$$

$$A' = \frac{(x^2+1)(8) - 8x \cdot (2x)}{(x^2+1)^2} = \frac{8 - 8x^2}{(x^2+1)^2} = \frac{8(1-x^2)}{(x^2+1)^2} = 0 \rightarrow$$

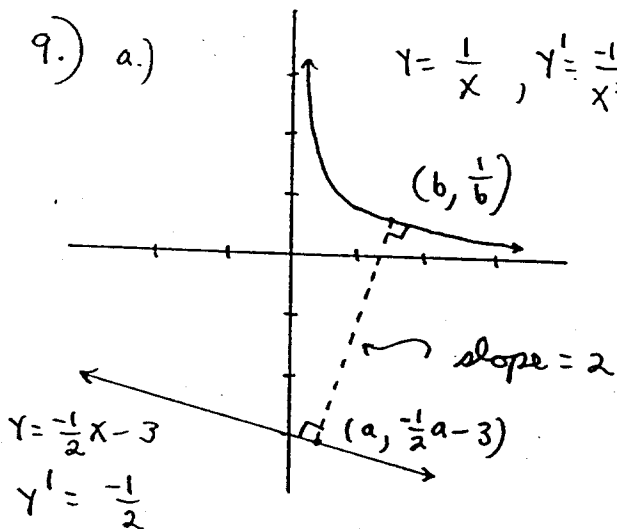
$$8(1-x^2) = 0 \rightarrow x=1$$

so rectangle is 2 by 2

$$\begin{array}{c} + \quad 0 \quad - \\ \hline x=1 \\ y=2 \end{array} \quad A'$$

$$\text{max. } A = 4$$

9.) a.)



b.) Determine a and b so that dotted line has slope 2:

$$b^2 = 2 \quad \text{and}$$

$$\frac{\frac{1}{b} - (-\frac{1}{2}a - 3)}{b - a} = 2 \rightarrow$$

$$b = \sqrt{2} \quad \text{and}$$

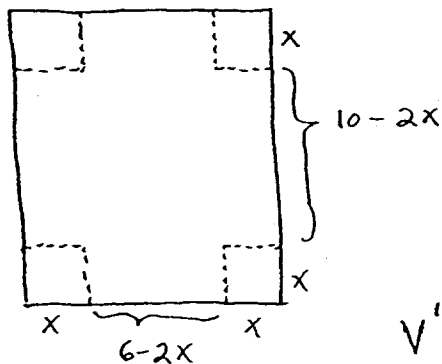
$$\frac{1}{\sqrt{2}} + \frac{1}{2}a + 3 = 2(\sqrt{2} - a) \rightarrow a = \frac{2}{5} \left(\frac{3}{\sqrt{2}} - 3 \right), \quad \text{so}$$

$$(b, \frac{1}{b}) = (\sqrt{2}, \frac{1}{\sqrt{2}}) \quad \text{and} \quad (a, \frac{1}{2}a - 3) = (-.35, -2.82) \quad \text{and}$$

distance between graphs is

$$L = \sqrt{(-2.82 - \frac{1}{\sqrt{2}})^2 + (-.35 - \sqrt{2})^2} \approx 3.94$$

10.)



maximize volume

$$V = x(6-2x)(10-2x)$$

$$= (6x-2x^2)(10-2x) \rightarrow$$

$$V' = (6x-2x^2)(-2) + (6-4x)(10-2x)$$

$$= -12x + 4x^2 + 60 - 12x - 40x + 8x^2$$

$$= 12x^2 - 64x + 60$$

$$= 4(3x^2 - 16x + 15) = 0 \rightarrow$$

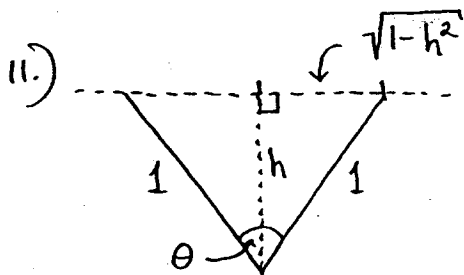
$$x = \frac{16 \pm \sqrt{16^2 - 12(15)}}{6} = \frac{16 \pm 2\sqrt{19}}{6} = \frac{8 \pm \sqrt{19}}{3} \rightarrow$$

$$x = 4.12 \text{ ft.}$$

$$x = 1.21 \text{ ft.}$$

↑ impossible

$$V' \quad \begin{array}{c} + \quad 0 \quad - \\ \hline x = 1.21 \text{ ft.} \end{array}, \quad \text{max. } V = 32.8 \text{ ft.}^3$$



maximize flow through gutter \rightarrow maximize area of triangle \rightarrow

$$A = h\sqrt{1-h^2} \rightarrow A' = h \cdot \frac{1}{2}(1-h^2)^{-\frac{1}{2}}(-2h) + \sqrt{1-h^2}$$

$$= \frac{-h^2}{\sqrt{1-h^2}} + \frac{\sqrt{1-h^2}}{1} = \frac{1-2h^2}{\sqrt{1-h^2}} = 0 \rightarrow h = \frac{1}{\sqrt{2}} \rightarrow \theta = \frac{\pi}{2}$$

$$A' \quad \begin{array}{c} 0 \\ \hline h = \frac{1}{\sqrt{2}} \text{ ft.} \approx 0.71 \text{ ft.}, \quad \theta = \frac{\pi}{2}, \text{ and} \\ \text{max. } A = \frac{1}{2} \text{ ft.}^2 \end{array}$$