

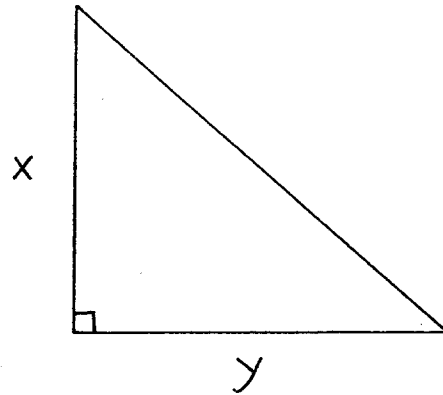
ESP  
Kouba  
Worksheet 14

1. For what values of  $x$  is  $f'(x) = 0$  ?

a.  $f(x) = x^3 (3 - 5x)^2$

b.  $f(x) = x \sqrt{x - 1}$

c.  $f(x) = \frac{x^3}{x^2 - 1}$



2. Leg  $x$  of the given right triangle is decreasing at the rate of 5 in./sec. and leg  $y$  is increasing at the rate of 7 in./sec. At what rate is the triangle's

- a.) hypotenuse
- b.) perimeter
- c.) area

changing when  $x = 4$  in. and  $y = 3$  in. ?

3. The perimeter of a square is increasing at the rate of 24 cm./min. At what rate is the edge  $x$  of the square changing when  $x = 10$  cm. ?

4. A balloon sits 8 ft. away from the base of a street light 18 ft. high. The balloon begins rising vertically at the constant rate of 3 ft./sec.

a.) How fast is the balloon's shadow moving away from the base of the street light when time  $t = 4$  seconds ?

b.) At what rate is the distance between the balloon and the the light (at the top of the pole) changing when time i.)  $t = 4$  sec. ? ii.)  $t = 6$  sec. ?

iii.)  $t = 8$  sec. ?

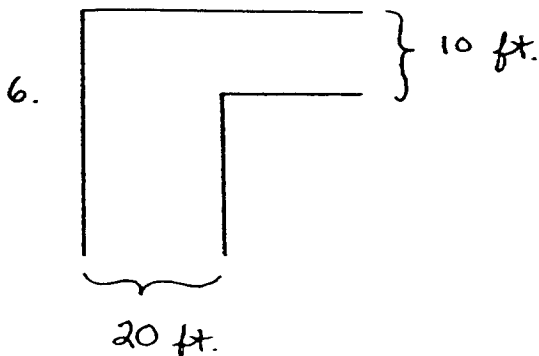
c.) What happens to the rate in part b.) as  $t \rightarrow \infty$  ?

5. The surface area of a sphere is decreasing at the rate of  $2\pi$  ft.<sup>2</sup>/sec.

a.) At what rate is the radius changing when the radius  $r = 4$  ft. ?

b.) At what rate is the volume changing when the radius  $r = 2$  ft. ?

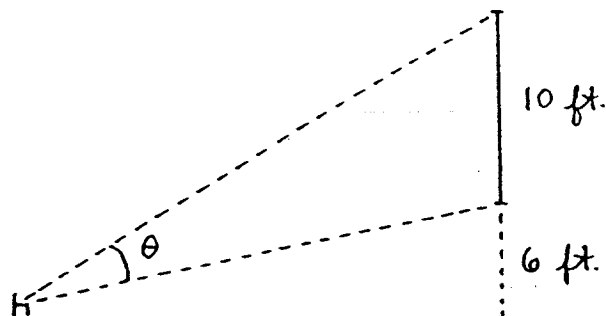
c.) For what radius  $r$  will the volume of the sphere be decreasing at the rate of  $6\pi$  ft.<sup>3</sup>/sec. ?



What is the length of the longest plank which can be carried horizontally around the corner of the corridor given in the diagram ?

7. If 100 apple trees are planted in a one acre orchard, at maturity each tree produces 55 bushels of apples. For each additional seven (7) trees planted in the orchard, the production per tree is reduced by two bushels. How many trees should be planted in the orchard to achieve the largest possible production of apples? What is the maximum production?

8. The movie screen in your local theater is 10 feet high and positioned 6 feet above the floor. How far away from the front of the theater should you sit in order to have the best view? (HINT: Find the largest possible  $\theta$  in the given diagram.)



9. A thin wire one meter long is to be cut into two pieces. One piece will be bent to form a square and the other a circle. How should the wire be cut in order that the sum of the areas of the circle and square be a maximum?

10. On the Fourth of July a fireworks charge is shot straight up from ground level so that it will explode into a colorful spectacle as it reaches its highest point. If the charge leaves the ground at 100 miles per hour, how long will it take to reach its highest point? How high above the ground will it be as it explodes?

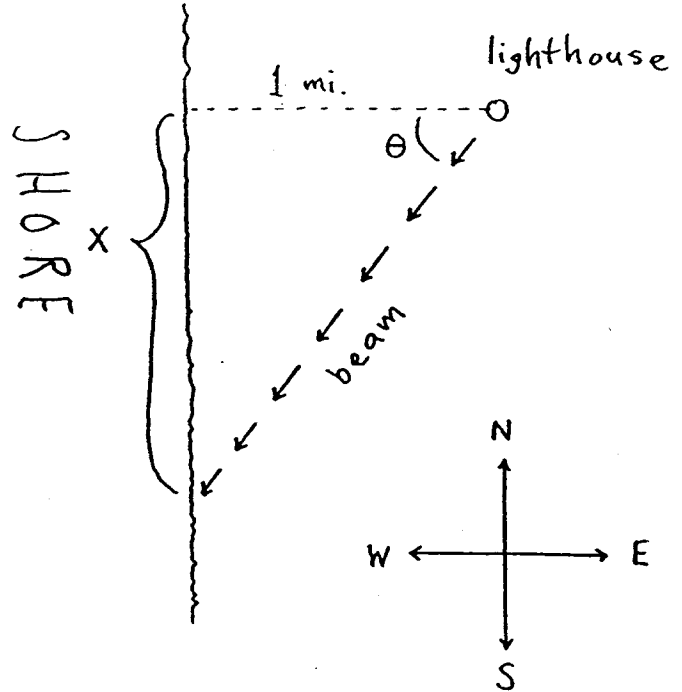
11. A lighthouse sits one mile offshore with a light beam turning counter-clockwise at the rate of ten revolutions per minute. See diagram on next page.

- Write distance  $x$  as a function of  $\theta$ .
- Assume that both  $x$  and  $\theta$  are functions of time  $t$ .

- i. Determine  $d\theta/dt$ .
- ii. Determine  $dx/dt$ .

c. How fast is the light beam racing down the shore line when the beam strikes the nearest point on the shore ?

d. How fast is the light beam racing down the shore line when the beam strikes a point twelve miles south of the nearest point on the shore ?



12. An egg is dropped from 256 feet directly above a small toy rocket which simultaneously begins rising vertically at the constant rate of 96 ft./sec.

- a.) In how many seconds will they collide ?
- b.) At what rate is the distance between the egg and the rocket changing at the moment they collide ?