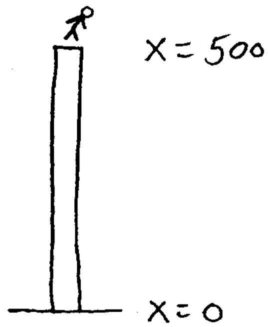
ESP
Kouba
Worksheet 15 Solutions

1.)



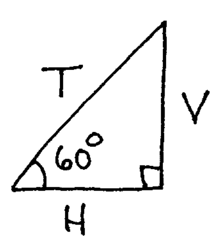
$x''(t) = -32 \rightarrow$
$x'(t) = -32t + c$
$x'(0) = -60$ mph $= -88$ ft./sec. $\rightarrow$
$\boxed{x'(t) = -32t - 88} \rightarrow$
$x(t) = -16t^2 - 88t + c$
$x(0) = 500 \rightarrow$
$\boxed{x(t) = -16t^2 - 88t + 500}$;

to hit ground : $x(t) = 0 \rightarrow -16t^2 - 88t + 500 = 0 \rightarrow$

$t = \dfrac{88 \pm \sqrt{39744}}{-32} =$ 3.48 sec. and

$x'(3.48) = -199.36$ ft./sec. $=$ -135.9 mph

2.)



Let T be initial velocity of rocket and
V : "vertical component" of T
H : "horizontal component" of T ;

analyze vertical motion of rocket :

acc. $s''(t) = -32$ ft./sec.$^2 \rightarrow$
$s'(t) = -32t + c$ (Since rocket travels for 15 seconds, it will be at its highest point in 7.5 seconds, i.e. $s'(7.5) = 0$!
$\rightarrow 0 = -32(7.5) + c \rightarrow c = 240$) $\rightarrow$
vel. $\underline{s'(t) = 240 - 32t}$; then

$V = s'(0) = 240$ ft./sec and

$\sin 60° = V/T \to T = \dfrac{V}{\sin 60°} = \dfrac{240}{\sqrt{3}/2} \to$

initial velocity $T \approx 277.13$ ft./sec. ;

$\cos 60° = \dfrac{H}{T} \to H = T\cos 60° \approx (277.13)(\tfrac{1}{2}) \approx 138.6$ ft./s.

so rocket travels $(138.6)(15) = 2078.46$ ft. ;

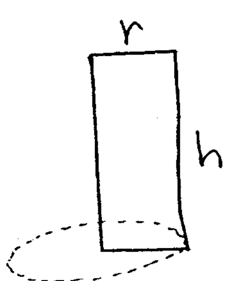
since $s'(t) = 240 - 32t \to s(t) = 240t - 16t^2 + c$

$(s(0) = 0 \to 0 = 0 + c \to c = 0) \to$ ht. $s(t) = 240t - 16t^2$

and so maximum height is

$s(7.5) = 240(7.5) - 16(7.5)^2 = 900$ ft. .

3)



$2r + 2h = 24$ in. $\to h = 12 - r$

maximize volume

$V = \pi r^2 h = \pi r^2(12 - r)$

$= 12\pi r^2 - \pi r^3 \to$

$V' = 24\pi r - 3\pi r^2 = 3\pi r(8 - r) = 0 \to r = 8$ in. $h = 4$ in.

$\begin{array}{ccc} + & 0 & - \end{array} \; V'$

$r = 8$ in.

max.

$V = 256\pi$ in.^{3}

4.) $X + Y = 1 \to Y = 1 - X$ and let

$S = X^2 + Y^3 = X^2 + (1-X)^3 \to S' = 2X - 3(1-X)^2$

$= -3x^2 + 8x - 3 = 0 \rightarrow x = \dfrac{-8 \pm \sqrt{64-36}}{-6} = \dfrac{-8 \pm 2\sqrt{7}}{-6}$

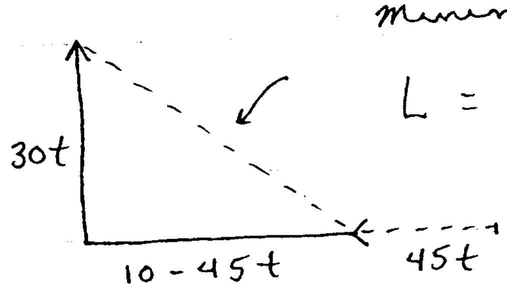
$= \cancel{2.21}$ or $.45$ and $y = .55 \rightarrow$

	$-$	0	$+$		S'
$x=0$		$x=.45$		$x=1$	
$y=1$		$y=.55$		$y=0$	
$\uparrow$		$\uparrow$		$\uparrow$	
max.		min.		max	
$S=1$		$S=0.369$		$S=1$	

a.) max : #'s are 0 and 1

b.) min : #'s are .45 and .55

5.) minimize length



$L = \sqrt{(30t)^2 + (10-45t)^2} \rightarrow$

$L' = \frac{1}{2}(\sim)^{-\frac{1}{2}} \cdot \{1800t + 2(10-45t)(-45)\} = 0 \rightarrow$

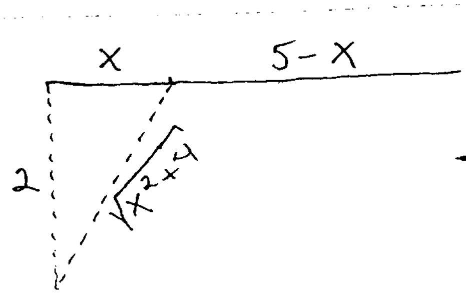
$5850t - 900 = 0 \rightarrow t = .1538$ hr. and

$-\quad 0 \quad +$ L'

$t = 0.1538$ hr.

$L = \sqrt{30.77} = 5.55$ miles.

6.) minimize time



$T = T_{swim} + T_{jog}$

$= \dfrac{\sqrt{x^2+4}}{2} + \dfrac{5-x}{4} \rightarrow$

$$T' = \frac{1}{2}\cdot\frac{1}{\cancel{2}}(x^2+4)^{-1/2}\cdot \cancel{2}x - \frac{1}{4} = \frac{x}{2\sqrt{x^2+4}} - \frac{1}{4} = 0 \rightarrow$$

$$4x = 2\sqrt{x^2+4} \rightarrow 4x^2 = x^2+4 \rightarrow x^2 = 4/3 \rightarrow$$

$x = \frac{2}{\sqrt{3}}$ miles.

0 T'

$x = 2/\sqrt{3}$ mi

min. $T \approx 2.12$ hr.

7.) $y'' = 1 - x^2 \rightarrow$

$y' = x - \frac{1}{3}x^3 + c$ and $y'(1) = -1$ (slope)

$-1 = 1 - \frac{1}{3} + c \rightarrow c = -5/3 \rightarrow$

$$y' = x - \frac{1}{3}x^3 - 5/3 \rightarrow$$

$y = \frac{1}{2}x^2 - \frac{1}{12}x^4 - \frac{5}{3}x + c$ and $y(1) = 1$

$1 = \frac{1}{2} - \frac{1}{12} - \frac{5}{3} + c \rightarrow c = \frac{9}{4} \rightarrow$

$$y = \frac{1}{2}x^2 - \frac{1}{12}x^4 - \frac{5}{3}x + \frac{9}{4}.$$