

ESP

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Worksheet 3 Solutions

$$1.) \text{ a.) } \lim_{x \rightarrow 1} \frac{(x-1)(x+3)}{x-1} = 4$$

$$\text{b.) } \lim_{x \rightarrow +\infty} \frac{1 - \frac{3}{x}}{2 + \frac{1}{x} + \frac{7}{x^2}} = \frac{1}{2}$$

$$\text{c.) } \lim_{x \rightarrow 1^+} \frac{x}{x-1} = \frac{1}{0^+} = +\infty \text{ (does not exist)}$$

$$\text{d.) } \lim_{x \rightarrow -\infty} \frac{x+7}{1 + \frac{13}{x}} = \frac{-\infty}{1} = -\infty \text{ (does not exist)}$$

$$\text{e.) } \lim_{x \rightarrow 1} \frac{\sqrt{x}-1}{(\sqrt{x}-1)(\sqrt{x}+1)} = \frac{1}{2}$$

$$\text{f.) } \lim_{x \rightarrow +\infty} \frac{1 - \frac{100,000}{x}}{x^4 + \frac{1}{x}} = \frac{1}{\infty} = 0$$

$$\text{g.) } \lim_{x \rightarrow +\infty} (1 + \cos x) \text{ does not exist since}$$

$$\lim_{x \rightarrow +\infty} \cos x \text{ does not exist.}$$

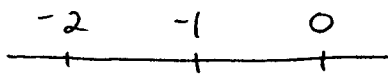
$$\text{h.) } \lim_{x \rightarrow -\infty} (\pi - \arctan x) = \pi - \left(-\frac{\pi}{2}\right) = \frac{3\pi}{2}$$

$$\text{i.) } \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{x^2(1 + \frac{1}{x})}} = \lim_{x \rightarrow 0^+} \frac{x}{\sqrt{x^2} \sqrt{1 + \frac{1}{x}}}$$

$$= \lim_{x \rightarrow 0^+} \frac{x}{x \sqrt{1 + \frac{1}{x}}} = \frac{1}{\infty} = 0$$

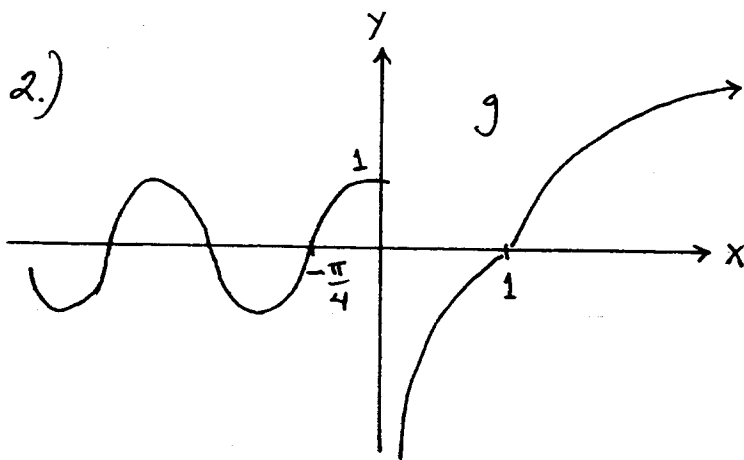
$$\text{j.) } \lim_{x \rightarrow 2} \frac{(x^2-4)(x^2+4)}{x-2} = \lim_{x \rightarrow 2} \frac{(x-2)(x+2)(x^2+4)}{x-2}$$

$$= (4)(8) = 32$$



$$\begin{aligned}
 \text{k.) } \lim_{x \rightarrow -1^+} \frac{|x^2 - 1|}{x^2 - 1} &= \lim_{x \rightarrow -1^+} \frac{-(x^2 - 1)}{x^2 - 1} = -1 \quad \text{and} \\
 \lim_{x \rightarrow -1^-} \frac{|x^2 - 1|}{x^2 - 1} &= \lim_{x \rightarrow -1^-} \frac{x^2 - 1}{x^2 - 1} = +1 \quad \text{so} \\
 \lim_{x \rightarrow -1} \frac{|x^2 - 1|}{x^2 - 1} &\text{ does not exist.}
 \end{aligned}$$

$$\begin{aligned}
 \text{l.) } \lim_{x \rightarrow +\infty} \frac{(\sqrt{x^2 + 1} - x)(\sqrt{x^2 + 1} + x)}{\sqrt{x^2 + 1} + x} &= \lim_{x \rightarrow +\infty} \frac{x^2 + 1 - x^2}{\sqrt{x^2 + 1} + x} \\
 &= \frac{1}{\infty} = 0
 \end{aligned}$$



$$\text{a.) } \lim_{x \rightarrow 0^-} g(x) = 1$$

$$\text{b.) } \lim_{x \rightarrow 0^+} g(x) = -\infty$$

$$\text{c.) } \lim_{x \rightarrow 1^-} g(x) = 0$$

$$\text{d.) } \lim_{x \rightarrow 1^+} g(x) = 0$$

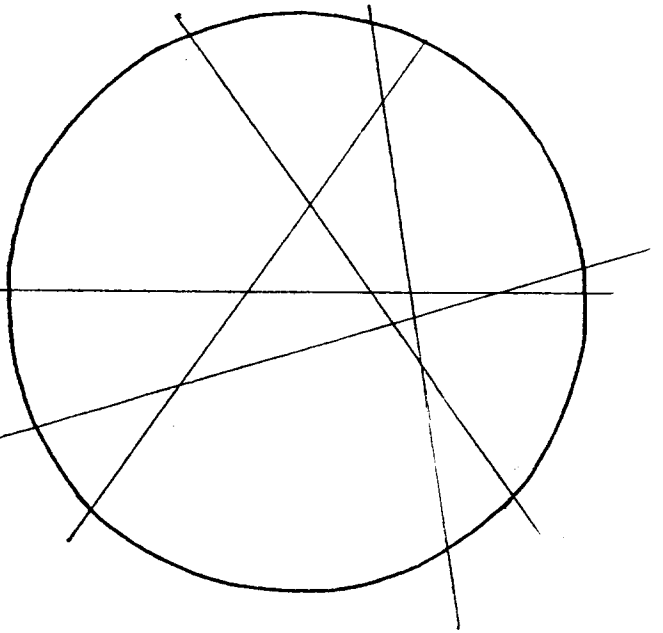
$$\text{e.) } \lim_{x \rightarrow -\infty} g(x) \text{ does not exist}$$

$$\text{f.) } \lim_{x \rightarrow +\infty} g(x) = +\infty$$

$$\begin{aligned}
 \text{3.) } Y &= x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x+2)^2 - 1 \rightarrow \\
 Y+1 &= (x+2)^2 \quad \text{for } x \leq -2 \rightarrow \\
 \sqrt{Y+1} &= \sqrt{(x+2)^2} = -(x+2) \rightarrow
 \end{aligned}$$

$-\sqrt{Y+1} = X+2 \rightarrow X = -2 - \sqrt{Y+1}$. Then
the inverse function $g(x) = -2 - \sqrt{x+1}$ works.

4.) <u>lines</u>	<u>pieces</u>
1	$2 = 1+(1)$
2	$4 = 1+(1+2)$
3	$7 = 1+(1+2+3)$
4	$11 = 1+(1+2+3+4)$
5	$16 = 1+(1+2+3+4+5)$
6	$22 = 1+(1+2+3+4+5+6)$
⋮	⋮
n	$1 + (1+2+3+4+\dots+n)$ $= 1 + \frac{n(n+1)}{2}$



$n = 20$ lines $\rightarrow 1 + \frac{20(21)}{2} = 211$ parts

5.) $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \dots}}} = X$ or $1 + \frac{1}{X} = X \rightarrow$

$X + 1 = X^2 \rightarrow 0 = X^2 - X - 1 \rightarrow$

$X = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2} = \frac{1 + \sqrt{5}}{2} \approx 1.618$

(This number is called the Golden Ratio)

$$b.) \ a.) \ \lim_{x \rightarrow -1} \frac{(x+1)(x+5)}{(x+1)(x^2-x+1)} = \frac{4}{3}$$

$$b.) \ \lim_{x \rightarrow 5^+} \frac{5x}{\sqrt{x-5}} = \frac{25}{0^+} = +\infty$$

$$c.) \ \lim_{x \rightarrow 0^+} \frac{\sin x}{\sqrt{x}} \cdot \frac{\sqrt{x}}{\sqrt{x}} = \lim_{x \rightarrow 0^+} \frac{\sin x}{x} \cdot \sqrt{x} = 1 \cdot 0 = 0$$

$$d.) \ \lim_{x \rightarrow 4} \frac{(\sqrt{x})^3 - 2^3}{x - 4} = \lim_{x \rightarrow 4} \frac{(\sqrt{x}-2)((\sqrt{x})^2 + 2\sqrt{x} + 4)}{(\sqrt{x}-2)(\sqrt{x}+2)} = \frac{12}{4} = 3$$

$$e.) \ \lim_{x \rightarrow +\infty} \frac{\sqrt{25x^2 + 3x + 5}}{4x} = \lim_{x \rightarrow +\infty} \frac{\sqrt{25x^2 + 3x + 5}}{\sqrt{16x^2}}$$

$$= \lim_{x \rightarrow +\infty} \sqrt{\frac{25}{16} + \frac{3}{16x} + \frac{5}{16x^2}} = \sqrt{\frac{25}{16}} = \frac{5}{4}$$

$$f.) \ \lim_{x \rightarrow -\infty} \frac{\sqrt{25x^2 + 3x + 5}}{-\sqrt{16x^2}} = \lim_{x \rightarrow -\infty} -\sqrt{\frac{25}{16} + \frac{3}{16x} + \frac{5}{16x^2}} = \frac{-5}{4}$$

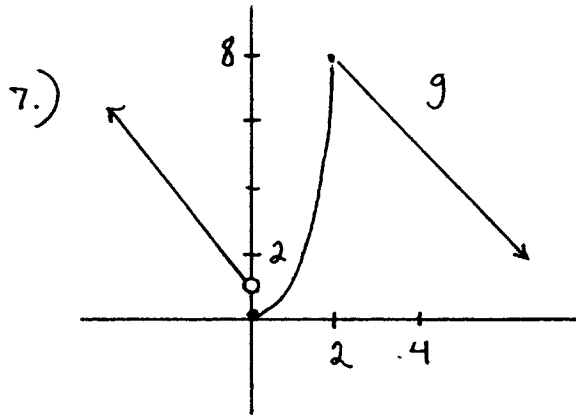
$$g.) \ \lim_{x \rightarrow 0} \frac{2x}{\sin x - x} = \lim_{x \rightarrow 0} \frac{2}{\frac{\sin x}{x} - 1} = \frac{2}{-0} = -\infty$$

$$h.) \ \lim_{x \rightarrow \frac{\pi}{2}^+} \tan x = -\infty$$

$$i.) \ \lim_{x \rightarrow 0^+} \sin \frac{1}{x} = \lim_{x \rightarrow +\infty} \sin x \text{ does not exist}$$

$$j.) \ \lim_{x \rightarrow 8} \frac{x^{\frac{1}{3}} - 2}{(x^{\frac{1}{3}})^3 - 2^3} = \lim_{x \rightarrow 8} \frac{x^{\frac{1}{3}} - 2}{(x^{\frac{1}{3}} - 2)((x^{\frac{1}{3}})^2 + 2x^{\frac{1}{3}} + 2^2)}$$

$$= \frac{1}{4 + 4 + 4} = \frac{1}{12}$$



a.) $\lim_{x \rightarrow 0^+} g(x) = 0$

b.) $\lim_{x \rightarrow 0^-} g(x) = 1$

c.) $\lim_{x \rightarrow 2^+} g(x) = 8$

d.) $\lim_{x \rightarrow 2^-} g(x) = 8$

e.) $\lim_{x \rightarrow 0} g(x)$ does not exist

f.) $\lim_{x \rightarrow 2} g(x) = 8$

g.) $\lim_{x \rightarrow +\infty} g(x) = -\infty$

h.) $\lim_{x \rightarrow -\infty} g(x) = +\infty$