ESP
Kouba
Worksheet 3 Solutions

1.) a.) $\lim_{x \to 1} \frac{\cancel{(x-1)}(x+3)}{\cancel{x-1}} = 4$

b.) $\lim_{x \to +\infty} \frac{1 - 3/x}{2 + \frac{1}{x} + \frac{7}{x^2}} = \frac{1}{2}$

c.) $\lim_{x \to 1^+} \frac{x}{x-1} = \frac{1}{0^+} = +\infty$ (does not exist)

d.) $\lim_{x \to -\infty} \frac{x+7}{1 + 13/x} = \frac{-\infty}{1} = -\infty$ (does not exist)

e.) $\lim_{x \to 1} \frac{\cancel{\sqrt{x}-1}}{\cancel{(\sqrt{x}-1)}(\sqrt{x}+1)} = \frac{1}{2}$

f.) $\lim_{x \to +\infty} \frac{1 - \frac{100,000}{x}}{x^4 + \frac{1}{x}} = \frac{1}{\infty} = 0$

g.) $\lim_{x \to +\infty} (1 + \cos x)$ does not exist since
$\lim_{x \to +\infty} \cos x$ does not exist.

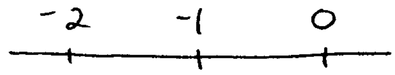
h.) $\lim_{x \to -\infty} (\pi - \arctan x) = \pi - \left(-\frac{\pi}{2}\right) = \frac{3\pi}{2}$

i.) $\lim_{x \to 0^+} \frac{x}{\sqrt{x^2(1+\frac{1}{x})}} = \lim_{x \to 0^+} \frac{x}{\sqrt{x^2}\sqrt{1+\frac{1}{x}}}$

$= \lim_{x \to 0^+} \frac{\cancel{x}}{\cancel{x}\sqrt{1+1/x}} = \frac{1}{\infty} = 0$

j.) $\lim_{x \to 2} \frac{(x^2-4)(x^2+4)}{x-2} = \lim_{x \to 2} \frac{\cancel{(x-2)}(x+2)(x^2+4)}{\cancel{x-2}}$

$= (4)(8) = 32$



k.) $\lim\limits_{x \to -1^+} \dfrac{|x^2-1|}{x^2-1} = \lim\limits_{x \to -1^+} \dfrac{-(\cancel{x^2-1})}{\cancel{x^2-1}} = -1$ and

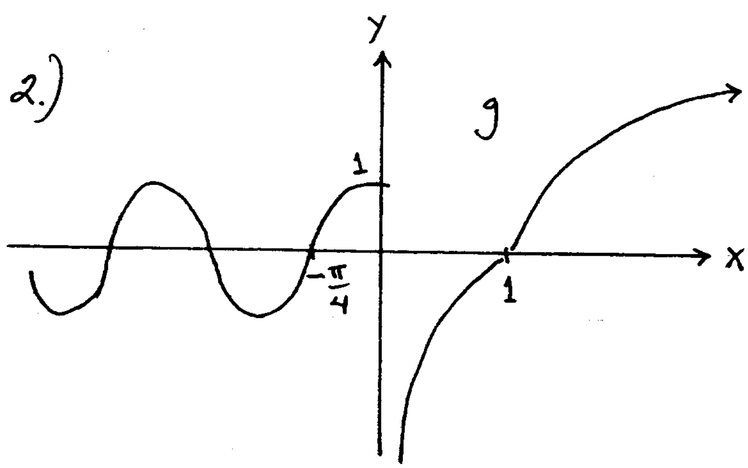
$\lim\limits_{x \to -1^-} \dfrac{|x^2-1|}{x^2-1} = \lim\limits_{x \to -1^-} \dfrac{\cancel{x^2-1}}{\cancel{x^2-1}} = +1$ so

$\lim\limits_{x \to -1} \dfrac{|x^2-1|}{x^2-1}$ does not exist.

l.) $\lim\limits_{x \to +\infty} \dfrac{\left(\sqrt{x^2+1} - x\right)\left(\sqrt{x^2+1} + x\right)}{\sqrt{x^2+1} + x} = \lim\limits_{x \to +\infty} \dfrac{\cancel{x^2}+1 - \cancel{x^2}}{\sqrt{x^2+1} + x}$

$= \dfrac{1}{\infty} = 0$

2.)



a.) $\lim\limits_{x \to 0^-} g(x) = 1$

b.) $\lim\limits_{x \to 0^+} g(x) = -\infty$

c.) $\lim\limits_{x \to 1^-} g(x) = 0$

d.) $\lim\limits_{x \to 1^+} g(x) = 0$ e.) $\lim\limits_{x \to -\infty} g(x)$ does not exist

f.) $\lim\limits_{x \to +\infty} g(x) = +\infty$

3.) $Y = x^2 + 4x + 3 = x^2 + 4x + 4 - 1 = (x+2)^2 - 1 \rightarrow$

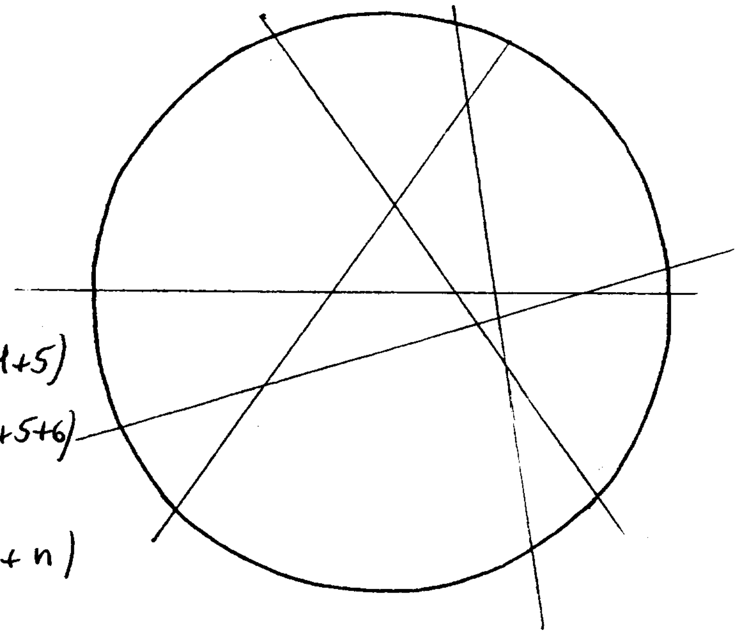
$Y + 1 = (x+2)^2$ for $x \le -2 \rightarrow$

$\sqrt{Y+1} = \sqrt{(x+2)^2} = -(x+2) \rightarrow$

$-\sqrt{Y+1} = X+2 \rightarrow X = -2-\sqrt{Y+1}$. Then the inverse function $g(x) = -2-\sqrt{x+1}$ works.

4.) <u>lines</u> <u>pieces</u>

lines	pieces
1	$2 = 1+(1)$
2	$4 = 1+(1+2)$
3	$7 = 1+(1+2+3)$
4	$11 = 1+(1+2+3+4)$
5	$16 = 1+(1+2+3+4+5)$
6	$22 = 1+(1+2+3+4+5+6)$
$\vdots$	$\vdots$
n	$1+(1+2+3+4+\cdots+n)$
	$= 1+\dfrac{n(n+1)}{2}$



$n = 20$ lines $\rightarrow 1+\dfrac{20(21)}{2} = 211$ parts

5.) $$1+\cfrac{1}{1+\cfrac{1}{1+\cfrac{1}{1+\ddots}}} = x \quad \text{or} \quad 1+\frac{1}{x} = x \rightarrow$$

$$x+1 = x^2 \rightarrow 0 = x^2-x-1 \rightarrow$$

$$x = \frac{-(-1) \pm \sqrt{(-1)^2-4(1)(-1)}}{2} = \frac{1+\sqrt{5}}{2} \approx 1.618$$

(This number is called the Golden Ratio)

6.) a.) $\lim_{x \to -1} \dfrac{\cancel{(x+1)}(x+5)}{\cancel{(x+1)}(x^2-x+1)} = \dfrac{4}{3}$

b.) $\lim_{x \to 5^+} \dfrac{5x}{\sqrt{x-5}} = \dfrac{25}{0^+} = +\infty$

c.) $\lim_{x \to 0^+} \dfrac{\sin x}{\sqrt{x}} \cdot \dfrac{\sqrt{x}}{\sqrt{x}} = \lim_{x \to 0^+} \dfrac{\sin x}{x} \cdot \sqrt{x} = 1 \cdot 0 = 0$

d.) $\lim_{x \to 4} \dfrac{(\sqrt{x})^3 - 2^3}{x - 4} = \lim_{x \to 4} \dfrac{\cancel{(\sqrt{x}-2)}((\sqrt{x})^2 + 2\sqrt{x} + 4)}{\cancel{(\sqrt{x}-2)}(\sqrt{x}+2)} = \dfrac{12}{4} = 3$

e.) $\lim_{x \to +\infty} \dfrac{\sqrt{25x^2+3x+5}}{4x} = \lim_{x \to +\infty} \dfrac{\sqrt{25x^2+3x+5}}{\sqrt{16x^2}}$

$= \lim_{x \to +\infty} \sqrt{\dfrac{25}{16} + \dfrac{3}{16x} + \dfrac{5}{16x^2}} = \sqrt{\dfrac{25}{16}} = \dfrac{5}{4}$

f.) $\lim_{x \to -\infty} \dfrac{\sqrt{25x^2+3x+5}}{-\sqrt{16x^2}} = \lim_{x \to -\infty} -\sqrt{\dfrac{25}{16} + \dfrac{3}{16x} + \dfrac{5}{16x^2}} = \dfrac{-5}{4}$

g.) $\lim_{x \to 0} \dfrac{2x}{\sin x - x} = \lim_{x \to 0} \dfrac{2}{\frac{\sin x}{x} - 1} = \dfrac{2}{-0} = -\infty$

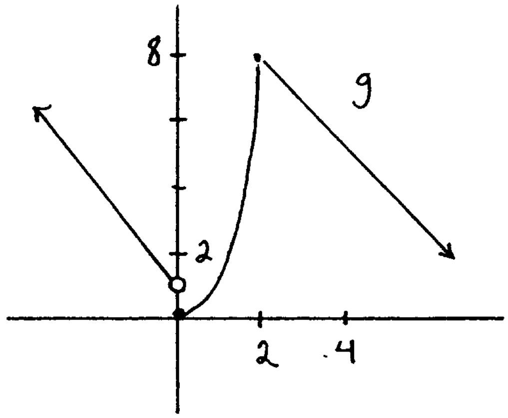
h.) $\lim_{x \to \frac{\pi}{2}^+} \tan x = -\infty$

i.) $\lim_{x \to 0^+} \sin \frac{1}{x} = \lim_{x \to +\infty} \sin x$ does not exist

j.) $\lim_{x \to 8} \dfrac{x^{1/3} - 2}{(x^{\frac{1}{3}})^3 - 2^3} = \lim_{x \to 8} \dfrac{\cancel{x^{\frac{1}{3}} - 2}}{\cancel{(x^{1/3}-2)}((x^{1/3})^2 + 2x^{1/3} + 2^2)}$

$= \dfrac{1}{4+4+4} = \dfrac{1}{12}$

7.)



a.) $\lim_{x\to 0^+} g(x) = 0$

b.) $\lim_{x\to 0^-} g(x) = 1$

c.) $\lim_{x\to 2^+} g(x) = 8$

d.) $\lim_{x\to 2^-} g(x) = 8$

e.) $\lim_{x\to 0} g(x)$ does not exist

f.) $\lim_{x\to 2} g(x) = 8$

g.) $\lim_{x\to +\infty} g(x) = -\infty$

h.) $\lim_{x\to -\infty} g(x) = +\infty$