

ESP
Kouba
Worksheet 6

1. Determine the following limits.

a. $\lim_{x \rightarrow 0} \frac{4x^2 + \sin 2x}{3x}$

b. $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x^2 - 9x}$

c. $\lim_{x \rightarrow \pi} \frac{1}{2 + 2 \cos x}$

d. $\lim_{x \rightarrow 3^-} \frac{(x-1)(x-2)}{(x+3)(x-3)}$

e. $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2 + 1}}{x + 2}$

f. $\lim_{x \rightarrow 0} \frac{x^2 \cos x + \sin 2x - 2x}{x^2 + 1}$

2. Sketch a rough graph for each of the following functions.

a. $y = x^3 - 1$

b. $y = \frac{1}{x^3 - 1}$

c. $y = (x + 3)^5 (x + 2)^3 x^2 (x - 1)^4 (x - 5)$

d. $y = \frac{(x-1)(x+2)^2}{(x+3)(x-4)^2}$

3. Determine the x-values for which the following functions are continuous.

a. $f(x) = x^3 + \sin 3x + |x|$

b. $f(x) = \tan x$

c. $f(x) = \frac{x-1}{x^2-1}$

d. $f(x) = \frac{1}{x^2-100}$

e. $f(x) = \begin{cases} \frac{x-2}{x^2-4} & \text{for } x \neq 2, -2 \\ 1/4 & \text{for } x = 2 \\ 1 & \text{for } x = -2 \end{cases}$

4. Determine the values of the constants m and n so that each of the following functions is continuous for all values of x , or state that this is impossible.

a. $f(x) = mx^3 + 7$

b. $f(x) = \frac{mx^2 + 1}{x - 5}$

c. $f(x) = \frac{m + x^2}{mx^2 - 1}$

d. $f(x) = \begin{cases} x^2 & \text{for } x \leq 2 \\ e^{mx} & \text{for } x > 2 \end{cases}$

e. $f(x) = \begin{cases} 0 & \text{for } x \leq 0 \\ \sqrt{x} & \text{for } 0 < x < 4 \\ m \cos[(x+1)\pi] & \text{for } x \geq 4 \end{cases}$

f. $f(x) = \begin{cases} 2mx + n & \text{for } x < 2 \\ 6 & \text{for } x = 2 \\ m - nx & \text{for } x > 2 \end{cases}$

g. $g(x) = \begin{cases} x + m & \text{for } x \leq 2 \\ \sqrt{mx^2 + 8} & \text{for } x > 2 \end{cases}$

h. $h(x) = \begin{cases} \frac{x^3 - x}{x^2 - 1} & \text{for } x \neq 1, -1 \\ m & \text{for } x = 1 \\ n & \text{for } x = -1 \end{cases}$

i. $g(x) = \begin{cases} \frac{x + 5}{x^2 + 3x - 10} & \text{for } x \neq 2, -5 \\ m & \text{for } x = 2 \\ n & \text{for } x = -5 \end{cases}$

5. One bucket contains 5 gallons of pineapple juice and another contains 5 gallons of kiwi juice. One cup of the kiwi juice is poured into the pineapple bucket and the mixture is stirred. One cup of this mixture is then poured back into the bucket of kiwi juice. Which of the following statements is true ?

a.) There is more pineapple juice in the kiwi bucket than kiwi juice in the pineapple bucket.

b. There is more kiwi juice in the pineapple bucket than pineapple juice in the kiwi bucket.

c. The amount of kiwi juice in the pineapple bucket is equal to the amount of pineapple juice in the kiwi bucket.

6. Verify with a careful and complete explanation that the equation $x^5 - x^2 + 17 = 2x$ has at least one solution.

7. Prove rigorously that the following statements about limits are true.

a. $\lim_{x \rightarrow 3} (2x + 1) = 7$

b. $\lim_{x \rightarrow +\infty} (1 + \sqrt{x})^3 - 100 = +\infty$

c. $\lim_{x \rightarrow 2} 1/x = 1/2$

d. $\lim_{x \rightarrow -1} x^2 = 1$