1. a. \( \lim_{x \to 0} \left( \frac{4}{3} x + \frac{2}{3} \cdot \frac{\sin 2x}{2x} \right) = 0 + \frac{2}{3} \cdot 1 = \frac{2}{3} \)

b. \( \lim_{x \to 0} \frac{\sin 2x}{2x} \cdot \frac{2}{(3x-9)} = 1 \cdot \frac{-2}{9} = -\frac{2}{9} \)

c. \( \lim_{x \to \pi} \frac{1}{2+2\cos x} = \frac{1}{0^+} = +\infty \)

d. \( \lim_{x \to 3^-} \frac{(x-1)(x-2)}{(x+3)(x-3)} = \frac{(2)(1)}{(6)(0^-)} = \frac{2}{0^-} = -\infty \)

e. \( \lim_{x \to -\infty} \frac{\sqrt{2x^2+1}}{x+2} = \lim_{x \to -\infty} \frac{\sqrt{2x^2+1}}{x+2} = -\sqrt{2} \)

f. \( \lim_{x \to 0} \frac{x^2 \cos x + \sin 2x - 2x}{x^2 + 1} = \frac{0 + 0 - 0}{0 + 1} = 0 \)

2. a. \( y = x^3 - 1 \)

b. \( y = \frac{1}{x^2 - 1} = \frac{1}{(x-1)(x+1)} \)

c. \( y = (x+3)(x+2)^3 x^2 (x-1)^4 (x-5) \)
3. a.) \( f \) is continuous for all \( x \)-values since \( y = x^3 \), \( y = \sin 3x \), and \( y = |x| \) are continuous for all \( x \)-values.

b.) \( f \) is continuous for all \( x \)-values except
\[ x = \frac{\pi}{2} \pm n\pi \] for
\[ n = 0, 1, 2, 3, \ldots \]

\[ y = \tan x \]

c.) \( f(x) = \frac{x-1}{(x-1)(x+1)} \) is continuous for all \( x \)-values except \( x = 1, x = -1 \).

d.) \( f(x) = \frac{1}{(x-10)(x+10)} \) is continuous for all \( x \)-values except \( x = 10, x = -10 \).

e.) \( f(x) = \begin{cases} \frac{x-2}{(x-2)(x+2)} & , x \neq 2, -2 \\ \frac{1}{4} & , x = 2 \\ 1 & , x = -2 \end{cases} = \begin{cases} \frac{1}{x+2} & , x \neq 2, -2 \\ \frac{1}{4} & , x = 2 \\ 1 & , x = -2 \end{cases} \)
4.) a) \( f(x) = mx^3 + 7 \) is a polynomial for all values of \( m \), and hence, is continuous for all values of \( x \) for all values of \( m \).

b) **Impossible**. For each value of \( m \), \( f \) is discontinuous at \( x = 5 \).

c) If \( m = 0 \), then \( f(x) = -x^2 \) is continuous.
   If \( m = -k < 0 \), then
   \[
   f(x) = \frac{-k + x^2}{-kx^2 - 1} = \frac{-(-k + x^2)}{kx^2 + 1}, \text{ where } kx^2 + 1 > 0.
   
   \[
   f(x) = \frac{m + x^2}{(mx - 1)(mx^2 + 1)}
   
   \text{is discontinuous at } x = \frac{1}{m} \text{ and } x = \frac{-1}{m}. \text{ Thus,}
   \]
   \[
   f \text{ is continuous for all } x \text{-values for } m \leq 0
   
   d) \quad \lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} x^2 = 4 \quad \text{and}
   \]
   \[
   \lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} e^{mx} = e^{2m}, \text{ so for}
continuity it must be that $4 = e^{2m} \Leftrightarrow \ln 4 = 2m \Leftrightarrow m = \frac{1}{2} \ln 4$.

e) $f$ is continuous at $x = 0$ since
$$
\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} 0 = 0 = \lim_{x \to 0^+} \sqrt{x} = \lim_{x \to 0^+} f(x),
$$
and
$$
\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \sqrt{x} = 2 \quad \text{and} \quad \lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} m \cos \left[(x+1)\pi\right] = m \cos 5\pi = -m.
$$
For continuity we need $2 = -m$ or $m = -2$.

f) $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (2mx + n) = 4m + n = 6$ and
$$
\lim_{x \to 2^+} f(x) = \lim_{x \to 2^+} (m - nx) = m - 2n = 6, \quad \text{then}
$$
$$
4m + n = 6 \quad \Rightarrow \quad 8m + 2n = 12 \quad \Rightarrow \quad 9m = 18 \Rightarrow m = 2 \quad \text{and} \quad n = -2.
$$

9) $\lim_{x \to 2^-} g(x) = \lim_{x \to 2^-} (x + m) = 2 + m$ and
$$
\lim_{x \to 2^+} g(x) = \lim_{x \to 2^+} \sqrt{mx^2 + 8} = \sqrt{4m + 8}, \quad \text{then}
$$
$$
2 + m = \sqrt{4m + 8} \Leftrightarrow (2 + m)^2 = 4m + 8 \Leftrightarrow 4 + 4m + m^2 = 4m + 8 \Leftrightarrow m^2 = 4 \Leftrightarrow m = 2 \text{ or } m = -2. \quad \text{But if } m = -2
then \( \sqrt{m^2 + 8} = \sqrt{-2x^2 + 8} = \sqrt{-2(x^2 - 4)} \) is not defined for \( x > 2 \). Thus, only \( m = 2 \) is a solution.

\[
\begin{align*}
\text{h.) } \lim_{x \to 1} h(x) &= \lim_{x \to 1} \frac{x^3 - x}{x^2 - 1} = \lim_{x \to 1} \frac{x(x - 1)(x + 1)}{(x - 1)(x + 1)} = 1 \\
\text{so let } \boxed{m = 1} \quad &\text{; } \lim_{x \to -1} h(x) = \lim_{x \to -1} \frac{x(x - 1)(x + 1)}{(x - 1)(x + 1)} = -1 \\
\text{so let } \boxed{n = -1} .
\end{align*}
\]

\[
\begin{align*}
\text{i.) } \lim_{x \to 2} g(x) &= \lim_{x \to 2} \frac{x + 5}{(x + 5)(x - 2)} = \frac{1}{0^+} \quad \text{so does not exist. Thus no choice for } m \text{ will make } g \text{ continuous at } x = 2 . \quad \underline{\text{Impossible}} .
\end{align*}
\]

(Note: \( n = \frac{1}{2} \) will make \( g \) continuous at \( x = -5 \).)
5) One cup of K.J. is added to 80 cups (5 gal.) of P.J. creating a mixture in the pineapple bucket which is \( \frac{80}{81} \) P.J. and \( \frac{1}{81} \) K.J. Thus, the one cup mixture returning to the kiwi bucket is

\[
\frac{80}{81} \text{ cup P.J.} \quad \text{and} \quad \frac{1}{81} \text{ cup K.J.}
\]

The amount of K.J. left behind in the pineapple bucket is

\[
1 \text{ cup} - \frac{1}{81} \text{ cup} = \frac{80}{81} \text{ cup K.J.}
\]

The amounts are equal.

6) Let \( f(x) = x^5 - x^2 - 2x + 17 \), which is continuous for all \( x \)-values since it is a polynomial. Consider \( f \) on the interval \([-2, 0]\) and let \( m = -15 \):

\[
f(-2) = -32 - 4 + 4 + 17 = -15 < 0 \quad \text{and} \quad f(0) = 17 > 0.
\]

Since \( m = 0 \) is between the values -15 and 17, by the I.M.T. there is at least one \( c \) in \([-2, 0]\) satisfying \( f(c) = 0 \), i.e., solving \( x^5 - x^2 - 2x + 17 = 0 \), i.e., solving \( x^5 - x^2 + 17 = 2x \)
7. a) Given any $\varepsilon > 0$, determine $\delta > 0$ so that if $0 < |x - 3| < \delta$, then $|f(x) - 7| < \varepsilon$, i.e.,

$$|(2x+1) - 7| = |2x - 6| = 2|x - 3| < \varepsilon$$

Choose $\delta = \varepsilon / 2$. If $0 < |x - 3| < \delta = \varepsilon / 2$, then $|x - 3| < \varepsilon / 2 \Rightarrow 2|x - 3| < \varepsilon$. This completes the proof.

b) Given any $E$, determine $D$ so that

if $x > D$, then $f(x) > E$, i.e.,

$$(1+\sqrt{x})^3 - 100 > E \Leftrightarrow (1+\sqrt{x})^3 > E + 100$$

$$\Leftrightarrow 1 + \sqrt{x} > (E+100)^{1/3}$$

$$\Leftrightarrow \sqrt{x} > 1 + (E+100)^{1/3}$$

$$\Leftrightarrow x > \left(1 + (E+100)^{1/3}\right)^2$$

Choose $D = \left(1 + (E+100)^{1/3}\right)^2$. This completes the proof.

c) Given any $\varepsilon > 0$, determine $\delta > 0$ so that if $0 < |x - 2| < \delta$, then $\left|\frac{1}{x} - \frac{1}{2}\right| < \varepsilon$, i.e.,

$$\left|\frac{2-x}{2x}\right| < \varepsilon$$

Assume that $\delta \leq 1$ so that $1 < \frac{1}{x} < 3$, $1 < |x| < 3$, and $\frac{1}{3} < \frac{1}{|x|} < 1$.

So that $\frac{1}{2} \frac{|x-2|}{|x|} \leq \frac{1}{2} \frac{|x-2|}{1} = \frac{1}{2} |x-2| < \varepsilon$. 

Choose $\delta = \text{minimum of } \frac{\varepsilon}{2}, 2\varepsilon^3$. If $0 < |x-2| < \delta$ then \[ \left| \frac{1}{x-2} - \frac{1}{2} \right| = \frac{1}{2} \frac{|x-2|}{|x|} \leq \frac{1}{2} |x-2| < \frac{1}{2} \cdot 2\varepsilon = \varepsilon. \]

This completes the proof.

d.) Given any $\varepsilon > 0$, determine $\delta > 0$ so that if $0 < |x-(-1)| = |x+1| < \delta$, then $|x^2 - 1| < \varepsilon$, i.e., \[ |(x-1)(x+1)| = |x-1||x+1| < \varepsilon. \]

Assume that $\delta \leq 1$ so that $-2 < x < 0$, $1 < |x-1| < 3$, and $|x-1||x+1| \leq 3|x+1| < \varepsilon$. Choose $\delta = \text{minimum of } \frac{\varepsilon}{2}, 2\varepsilon^3$. If $0 < |x+1| < \delta$ then $|x^2 - 1| = |x-1||x+1| \leq 3|x+1| < 3 \cdot \frac{\varepsilon}{3} = \varepsilon$. This completes the proof.