

1.) a.) $\lim_{x \rightarrow 0} \left(\frac{4}{3}x + \frac{2}{3} \cdot \frac{\sin 2x}{2x} \right) = 0 + \frac{2}{3} \cdot 1 = \frac{2}{3}$

b.) $\lim_{x \rightarrow 0} \frac{\sin 2x}{2x} \cdot \frac{2}{(3x-9)} = 1 \cdot \frac{-2}{9} = \frac{-2}{9}$

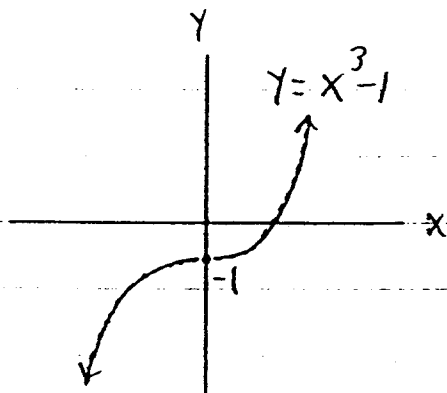
c.) $\lim_{x \rightarrow \pi} \frac{1}{2+2\cos x} = \frac{1}{0^+} = +\infty$

d.) $\lim_{x \rightarrow 3^-} \frac{(x-1)(x-2)}{(x+3)(x-3)} = \frac{(2)(1)}{(6)(0^-)} = \frac{2}{0^-} = -\infty$

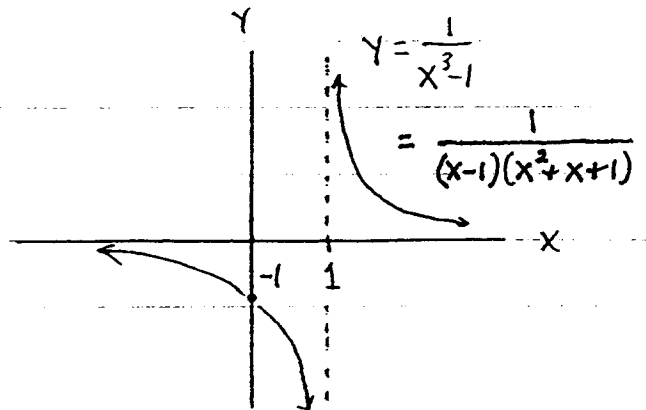
e.) $\lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2+1}}{x+2} = \lim_{x \rightarrow -\infty} \frac{\sqrt{2x^2+1}}{-\sqrt{(x+2)^2}}$
 $= \lim_{x \rightarrow -\infty} -\sqrt{\frac{2x^2+1}{x^2+4x+4}} = \lim_{x \rightarrow -\infty} -\sqrt{\frac{2 + \frac{1}{x^2}}{1 + \frac{4}{x} + \frac{4}{x^2}}} = -\sqrt{2}$

f.) $\lim_{x \rightarrow 0} \frac{x^2 \cos x + \sin 2x - 2x}{x^2 + 1} = \frac{0 + 0 - 0}{0 + 1} = 0$

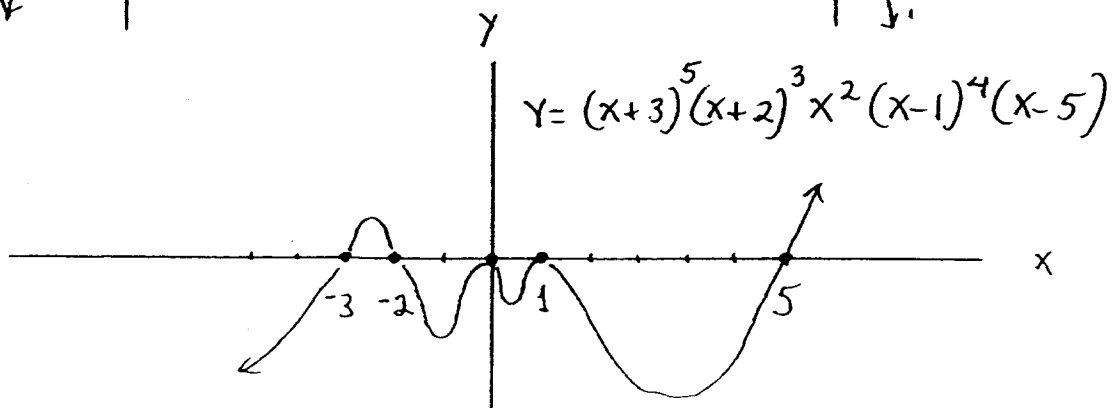
2.) a.)



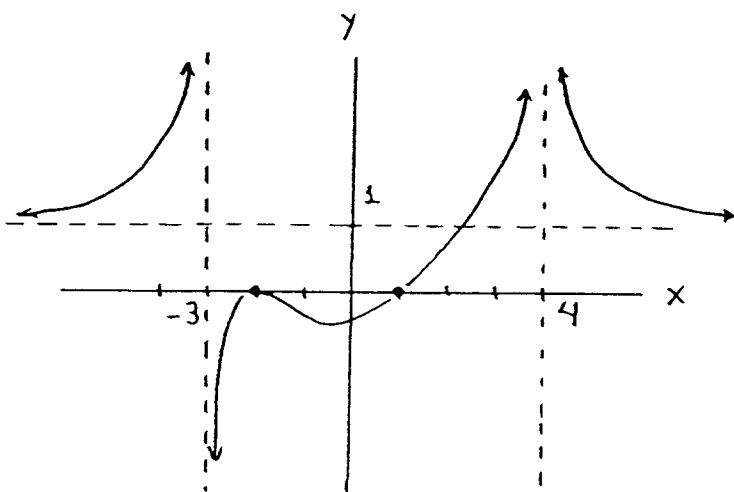
b.)



c.)

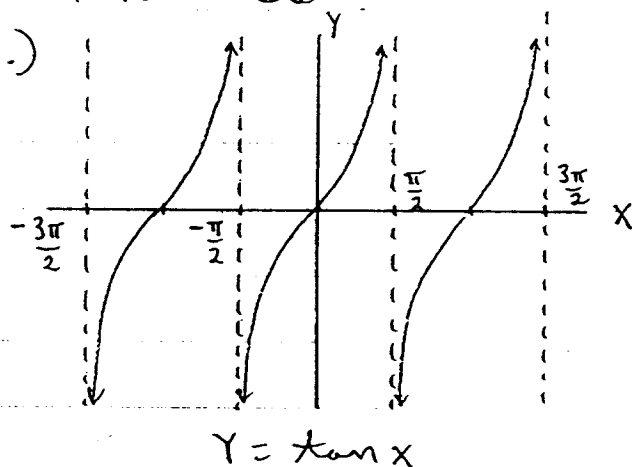


d.)



3.) a.) f is continuous for all x -values since $y = x^3$, $y = \sin 3x$, and $y = |x|$ are continuous for all x -values

b.)



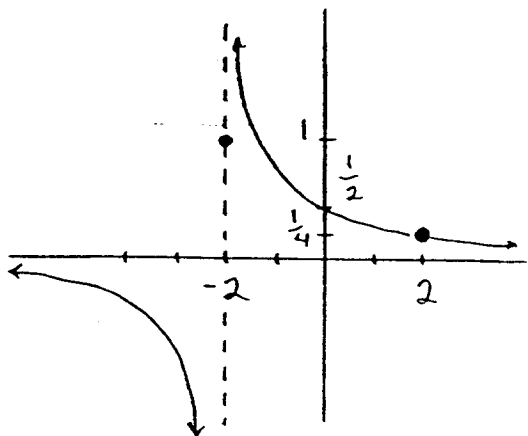
f is continuous for all x -values except

$$x = \frac{\pi}{2} \pm n\pi \text{ for } n = 0, 1, 2, 3, \dots$$

c.) $f(x) = \frac{x-1}{(x-1)(x+1)}$ is continuous for all x -values except $x=1, x=-1$.

d.) $f(x) = \frac{1}{(x-10)(x+10)}$ is continuous for all x -values except $x=10, x=-10$.

$$e.) f(x) = \begin{cases} \frac{x-2}{(x-2)(x+2)}, & x \neq 2, -2 \\ \frac{1}{4}, & x = 2 \\ 1, & x = -2 \end{cases} = \begin{cases} \frac{1}{x+2}, & x \neq 2, -2 \\ \frac{1}{4}, & x = 2 \\ 1, & x = -2 \end{cases}$$



f is continuous for all x -values except $x = -2$.

4.) a.) $f(x) = mx^3 + 7$ is a polynomial for all values of m , and, hence, is continuous for all values of x for all values of m .

b.) Impossible. For each value of m , f is discontinuous at $x = 5$.

c.) If $m = 0$, then $f(x) = -x^2$ is continuous
If $m = -k < 0$, then

$$f(x) = \frac{-k + x^2}{-kx^2 - 1} = \frac{-(-k + x^2)}{kx^2 + 1}, \text{ where } kx^2 + 1 > 0$$

If $m > 0$ then $f(x) = \frac{m + x^2}{(\sqrt{m}x - 1)(\sqrt{m}x + 1)}$ is

discontinuous at $x = \frac{1}{\sqrt{m}}$ and $x = -\frac{1}{\sqrt{m}}$. Thus,

f is continuous for all x -values for $\boxed{m \leq 0}$

d.) $\lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} x^2 = 4$ and

$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} e^{mx} = e^{2m}$, so for

continuity it must be that $4 = e^{2m} \Leftrightarrow \ln 4 = 2m \Leftrightarrow \underline{m = \frac{1}{2} \ln 4}$.

e.) f is continuous at $x=0$ since

$$\lim_{x \rightarrow 0^-} f(x) = \lim_{x \rightarrow 0^-} 0 = 0 = \lim_{x \rightarrow 0^+} \sqrt{x} = \lim_{x \rightarrow 0^+} f(x);$$

$$\text{and } \lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^-} \sqrt{x} = 2 \text{ and}$$

$$\lim_{x \rightarrow 4^+} f(x) = \lim_{x \rightarrow 4^+} m \cos[(x+1)\pi] = m \cos 5\pi = -m.$$

For continuity we need $2 = -m$ or $\boxed{m = -2}$.

$$f.) \lim_{x \rightarrow 2^-} f(x) = \lim_{x \rightarrow 2^-} (2mx + n) = 4m + n = 6 \text{ and}$$

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} (m - nx) = m - 2n = 6, \text{ then}$$

$$\left. \begin{array}{l} 4m + n = 6 \\ m - 2n = 6 \end{array} \right\} \left. \begin{array}{l} 8m + 2n = 12 \\ m - 2n = 6 \end{array} \right\} 9m = 18 \rightarrow \boxed{m = 2} \text{ and } \boxed{n = -2}$$

$$g.) \lim_{x \rightarrow 2^-} g(x) = \lim_{x \rightarrow 2^-} (x + m) = 2 + m \text{ and}$$

$$\lim_{x \rightarrow 2^+} g(x) = \lim_{x \rightarrow 2^+} \sqrt{mx^2 + 8} = \sqrt{4m + 8}, \text{ then}$$

$$2 + m = \sqrt{4m + 8} \Leftrightarrow (2 + m)^2 = 4m + 8 \Leftrightarrow$$

$$4 + 4m + m^2 = 4m + 8 \Leftrightarrow m^2 = 4 \Leftrightarrow$$

$$m = 2 \text{ or } m = -2. \text{ But if } m = -2$$

then $\sqrt{mx^2+8} = \sqrt{-2x^2+8} = \sqrt{-2(x^2-4)}$ is not defined for $x > 2$. Thus, only $\boxed{m=2}$ is a solution.

$$h.) \quad \lim_{x \rightarrow 1} h(x) = \lim_{x \rightarrow 1} \frac{x^3 - x}{x^2 - 1} = \lim_{x \rightarrow 1} \frac{x \cancel{(x-1)}(x+1)}{\cancel{(x-1)}(x+1)} = 1$$

so let $\boxed{m=1}$; $\lim_{x \rightarrow -1} h(x) = \lim_{x \rightarrow -1} \frac{x \cancel{(x-1)}(x+1)}{\cancel{(x+1)}(x+1)} = -1$

so let $\boxed{n=-1}$.

$$i.) \quad \lim_{x \rightarrow 2} g(x) = \lim_{x \rightarrow 2} \frac{\cancel{x+5}}{(\cancel{x+5})(x-2)} = \frac{1}{0^\pm} \quad \text{so}$$

does not exist. Thus no choice for m will make g continuous at $x=2$. Impossible.

(Note: $n = \frac{-1}{7}$ will make g continuous at $x=-5$.)

5.) One cup of K.J. is added to 80 cups (5 gal.) of P.J. creating a mixture in the pineapple bucket which is $\frac{80}{81}$ P.J. and $\frac{1}{81}$ K.J. Thus, the one cup mixture returning to the kiwi bucket is

$$\boxed{\frac{80}{81} \text{ cup P.J.}} \text{ and } \frac{1}{81} \text{ cup K.}$$

The amount of K.J. left behind in the pineapple bucket is

$$1 \text{ cup} - \frac{1}{81} \text{ cup} = \boxed{\frac{80}{81} \text{ cup K.J.}}$$

The amounts are equal.

6.) Let $f(x) = x^5 - x^2 - 2x + 17$, which is continuous for all x -values since it is a polynomial. Consider f on the interval $[-2, 0]$ and let $m=0$:

$$f(-2) = -32 - 4 + 4 + 17 = -15 < 0 \text{ and}$$

$$f(0) = 17 > 0. \text{ Since } m=0 \text{ is between the values } -15 \text{ and } 17,$$

by the IMVT there is at least one

$\# c$ in $[-2, 0]$ satisfying $f(c) = 0$, i.e., solving $x^5 - x^2 - 2x + 17 = 0$, i.e., solving $x^5 - x^2 + 17 = 2x$.

7.) a.) Given any $\varepsilon > 0$, determine $\delta > 0$ so that if $0 < |x-3| < \delta$, then $|f(x) - 7| < \varepsilon$, i.e.,

$$|(2x+1) - 7| = |2x-6| = 2|x-3| < \varepsilon$$

Choose $\delta = \varepsilon/2$. If $0 < |x-3| < \delta = \varepsilon/2$, then $|x-3| < \varepsilon/2 \Rightarrow 2|x-3| < \varepsilon$. This completes the proof.

b.) Given any E , determine D so that if $x > D$, then $f(x) > E$, i.e.,

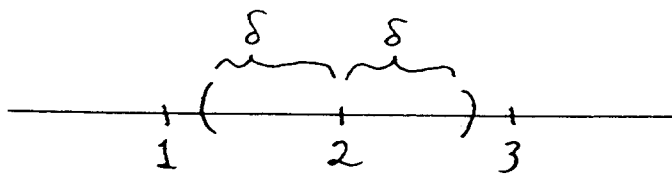
$$(1+\sqrt{x})^3 - 100 > E \Leftrightarrow (1+\sqrt{x})^3 > E+100$$

$$\Leftrightarrow 1+\sqrt{x} > (E+100)^{1/3}$$

$$\Leftrightarrow \sqrt{x} > 1 + (E+100)^{1/3}$$

$$\Leftrightarrow x > \{1 + (E+100)^{1/3}\}^2$$

Choose $D = \{1 + (E+100)^{1/3}\}^2$. This completes the proof.



c.) Given any $\varepsilon > 0$, determine $\delta > 0$ so that if $0 < |x-2| < \delta$, then $|\frac{1}{x} - \frac{1}{2}| < \varepsilon$, i.e.,

$$\left| \frac{2-x}{2x} \right| < \varepsilon, \text{ i.e., } \frac{1}{2} \frac{|x-2|}{|x|} < \varepsilon. \text{ Assume that}$$

$$\delta \leq 1 \text{ so that } 1 < x < 3, \quad 1 < |x| < 3, \quad \text{and } \frac{1}{3} < \frac{1}{|x|} < 1$$

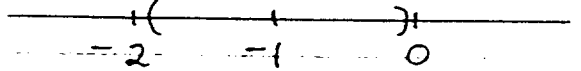
$$\text{so that } \frac{1}{2} \frac{|x-2|}{|x|} \leq \frac{1}{2} \frac{|x-2|}{|1|} = \frac{1}{2} |x-2| < \varepsilon.$$

Choose $\delta = \text{minimum of } \{1, 2\epsilon\}$. If $0 < |x-2| < \delta$
 then $\left| \frac{1}{x} - \frac{1}{2} \right| = \frac{1}{2} \frac{|x-2|}{|x|} \leq \frac{1}{2} \cdot |x-2| < \frac{1}{2} \cdot 2\epsilon = \epsilon$.

This completes the proof.

d.) Given any $\epsilon > 0$, determine $\delta > 0$ so that
 if $0 < |x - (-1)| = |x+1| < \delta$, then $|x^2 - 1| < \epsilon$, i.e.,
 $|(x-1)(x+1)| = |x-1||x+1| < \epsilon$.

Assume that $\delta \leq 1$



so that $-2 < x < 0$, $1 < |x-1| < 3$,

and $|x-1||x+1| \leq 3|x+1| < \epsilon$. Choose

$\delta = \text{minimum of } \{1, \epsilon/3\}$. If $0 < |x+1| < \delta$

then $|x^2 - 1| = |x-1||x+1| \leq 3|x+1| < 3 \cdot \frac{\epsilon}{3} = \epsilon$. This
 completes the proof.