1) a) \( f(x) = \sin x \) is continuous for all values of \( x \).

b) \( f(x) = \frac{1}{\sin x} \) is continuous for all values of \( x \) except where \( \sin x = 0 \), i.e., except \( x = 0, \pm \pi, \pm 2\pi, \pm 3\pi, \ldots \).

c) \( f(x) = \frac{x^4 - 1}{(x-1)(x+1)} \) is continuous everywhere except \( x = 1 \) and \( x = -1 \).

d) \( \lim_{x \to 1} f(x) = \lim_{x \to 1} \frac{(x^2 + 1)(x^2 - 1)}{(x^2 - 1)} = 2 = f(1) \)

and \( \lim_{x \to -1} f(x) = \lim_{x \to -1} \frac{(x^2 + 1)(x^2 - 1)}{(x^2 - 1)} = 2 \neq f(-1) \)

so \( f \) is continuous everywhere except at \( x = -1 \).

e) \( \lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} (x^2 + x) = 0 \) and

\( \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\sin x}{\sqrt{x}} = \lim_{x \to 0^+} \frac{\sqrt{x}}{\sqrt{x}} \cdot \lim_{x \to 0^+} \sin x = 0 \cdot 1 = 0 \); \( f \) is not defined at \( x = 2\pi \), so \( f \) is continuous for all \( x \)-values except \( x = 2\pi \).
2) a) i.) \( \text{Ave} = \frac{T(4) - T(1)}{4 - 1} = \frac{48 - 3}{3} = 15 \text{ mph} \)

ii.) \( \text{Ave} = \frac{T(2) - T(1)}{2 - 1} = \frac{12 - 3}{1} = 9 \text{ mph} \)

iii.) \( \text{Ave} = \frac{T(1.1) - T(1)}{1.1 - 1} = \frac{3.63 - 3}{0.1} = 6.3 \text{ mph} \)

iv.) \( \text{Ave} = \frac{T(1.01) - T(1)}{1.01 - 1} = \frac{3.0603 - 3}{0.01} = 6.03 \text{ mph} \)

b.) \( T' = \lim_{h \to 0} \frac{T(t+h) - T(t)}{h} = \lim_{h \to 0} \frac{3(t+h)^2 - 3t^2}{h} \)

\[ = \lim_{h \to 0} \frac{3(t^2 + 2th + h^2) - 3t^2}{h} = \lim_{h \to 0} \frac{3t^2 + 6th + 3h^2 - 3t^2}{h} \]

\[ = \lim_{h \to 0} \frac{h(6t + 3h)}{h} = 6t \text{ so velocity at } \]

\[ t = 1 \text{ is } T'(1) = 6(1) = 6 \text{ mph} \]

3) a) i.) \( \text{Ave} = \frac{\sqrt{3} - \sqrt{2}}{3 - 2} \approx 0.3178 \text{ gm./cm.} \)

ii.) \( \text{Ave} = \frac{\sqrt{2.5} - \sqrt{2}}{2.5 - 2} \approx 0.3338 \text{ gm./cm.} \)

iii.) \( \text{Ave} = \frac{\sqrt{2.1} - \sqrt{2}}{2.1 - 2} \approx 0.3492 \text{ gm./cm.} \)

iv.) \( \text{Ave} = \frac{\sqrt{2.01} - \sqrt{2}}{2.01 - 2} \approx 0.3531 \text{ gm./cm.} \)

b.) \( \text{Density} = \lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{(2+h) - 2} = \lim_{h \to 0} \frac{\sqrt{2+h} - \sqrt{2}}{h} \frac{\sqrt{2+h} + \sqrt{2}}{\sqrt{2+h} + \sqrt{2}} \)
\[ \lim_{h \to 0} \frac{(2 + h) - 2}{h (\sqrt{2+h} + \sqrt{2})} = \lim_{h \to 0} \frac{h}{h (\sqrt{2+h} + \sqrt{2})} = \frac{1}{2\sqrt{2}} \approx 0.3535 \text{ m/s} \]

4. a) i) \[ m = \frac{1}{3 - \frac{3}{2}} - \frac{1}{3 - \frac{3}{2}} = -1 \]

ii) \[ m = \frac{1}{2 - \frac{3}{2}} - \frac{1}{2 - \frac{3}{2}} = -2 \]

iii) \[ m = \frac{\frac{2}{4} - 1}{\frac{7}{4} - \frac{3}{2}} = -\frac{8}{3} = -2.6666 \]

iv) \[ m = \frac{\frac{25}{16} - 1}{\frac{25}{16} - \frac{3}{2}} = -\frac{32}{9} = -3.5555 \]

b) Slope = \[ \lim_{h \to 0} \frac{f(\frac{3}{2} + h) - f(\frac{3}{2})}{h} = \lim_{h \to 0} \frac{1}{h} \left( \frac{\frac{3}{2} + h - \frac{3}{2}}{h} - \frac{1}{\frac{3}{2} - 1} \right) \]

\[ = \lim_{h \to 0} \left( \frac{1}{\frac{3}{2} + h} - 2 \right) \cdot \frac{1}{h} = \lim_{h \to 0} \frac{-2h}{(\frac{1}{2} + h) h} = -4 \]

5. a) \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]

\[ = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{h - 2xh - h^2}{h} \]

\[ = \lim_{h \to 0} \frac{h(1 - 2x - h)}{h} = 1 - 2x \]
b) \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
\[ = \lim_{h \to 0} \frac{(x+h) + 1 - x + 1}{2(x+h) - 3} \cdot \frac{-5h}{(2x + 2h - 3)(2x - 3)} \cdot \frac{1}{h} \]
\[ = \frac{-5}{(2x - 3)^2} \]

c) \[ g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} \]
\[ = \lim_{h \to 0} \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \cdot \frac{\sqrt{x+h-5} - \sqrt{x-5}}{h} \cdot \frac{\sqrt{x+h-5} + \sqrt{x-5}}{\sqrt{x+h-5} + \sqrt{x-5}} \]
\[ = \lim_{h \to 0} \frac{(x+h-5) - (x-5)}{h(\sqrt{x+h-5} + \sqrt{x-5})} = \lim_{h \to 0} \frac{h}{x(\sqrt{x+h-5} + \sqrt{x-5})} = \frac{1}{2\sqrt{x-5}} \]

d) \[ H'(x) = \lim_{h \to 0} \frac{H(x+h) - H(x)}{h} \]
\[ = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} \cdot \frac{1}{h} \]
\[ = \lim_{h \to 0} \{h + \frac{1}{x^2} - \frac{1}{(x+h)^2}\} \cdot \frac{1}{h} \]
\[ = \lim_{h \to 0} \left\{h + \frac{2xh + h^2}{x^2(x+h)^2}\right\} \cdot \frac{1}{h} \]
\[ = 1 + \frac{2x}{x^2} = 1 + \frac{2}{x^3} \]

e) \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]
\[
\begin{align*}
\lim_{h \to 0} \frac{\cos 4(x+h) - \cos 4x}{h} &= \lim_{h \to 0} \cos(4x+4h) - \cos 4x \\
&= \lim_{h \to 0} \frac{\cos 4x \cdot \cos 4h - \sin 4x \cdot \sin 4h - \cos 4x}{h} \\
&= \lim_{h \to 0} \left\{ \frac{\cos 4x \cdot \cos 4h - 1}{4h} \cdot 4 - \sin 4x \cdot \frac{\sin 4h}{4h} \cdot 4 \right\} \\
&= \cos 4x \cdot (0) \cdot 4 - \sin 4x \cdot (1) \cdot 4 \\
&= -4 \sin 4x
\end{align*}
\]

\[f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h} = \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} = \lim_{h \to 0} \frac{2xh + h^2}{h} = 2x + \lim_{h \to 0} h = 2x \quad \text{for } x < 0
\]

\[f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{1}{2}(x+h) - \frac{1}{2}x}{h} = \lim_{h \to 0} \frac{\frac{1}{2}h}{h} = \frac{1}{2} = \frac{1}{2} \quad \text{for } x = 0
\]

\[f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h) - f(0)}{h} = \lim_{h \to 0} \frac{f(h) - 0}{h} = \lim_{h \to 0} \frac{f(h)}{h} \Rightarrow f(0) = 0
\]
\[
\begin{align*}
\lim_{h \to 0^+} \frac{f(h)}{h} &= \lim_{h \to 0^+} \frac{h^2}{h} = \lim_{h \to 0^+} h = 0 \\
\lim_{h \to 0^-} \frac{f(h)}{h} &= \lim_{h \to 0^-} \frac{-\frac{1}{2}h}{h} = \frac{1}{2}
\end{align*}
\]
so

\[f'(0) \text{ does not exist, i.e.,} \]

\[f'(x) = \begin{cases} 
2x & \text{for } x > 0 \\
\text{does not exist for } x = 0 \\
\frac{1}{2} & \text{for } x < 0
\end{cases}
\]

6.)

\[
Y = x^2 \\
\rightarrow Y' = 2x
\]

Slope of tangent line at \(x = a\) is

i.) \(\frac{a^2 - 2}{a - 3}\)

and ii.) \(2a\), thus

\[
\frac{a^2 - 2}{a - 3} = 2a \Rightarrow a^2 - 2 = 2a^2 - 6a \Rightarrow 0 = a^2 - 6a + 2
\]

\[
a = \frac{6 \pm \sqrt{36 - 8}}{2} = 3 \pm \sqrt{7}
\]
This graph suggests that there are only 2 solutions. However, the IMVT will prove that there are at least 3 solutions:

Let \( f(x) = x^2 - 1.9^x \), which is continuous for all values of \( x \) since \( y = x^2 \) and \( y = 1.9^x \) are continuous for all values of \( x \). Consider \( f \) on the interval \([-1, 0]\):

\[
f(-1) = 1 - \frac{1}{1.9} = \frac{9}{19} > 0 \quad \text{and} \quad f(0) = 0 - 1 = -1 < 0.
\]

By the IMVT there is at least one number \( c \) in \([-1, 0]\) satisfying \( f(c) = 0 \), i.e., solving the equation \( x^2 = 1.9^x \). Consider \( f \) on the interval \([0, 2]\):

\[
f(0) = 0 - 1 = -1 < 0 \quad \text{and} \quad f(2) = 4 - 1.9^2 = 0.39 > 0.
\]

By the IMVT there is at least one number \( c \) in \([0, 2]\) satisfying \( f(c) = 0 \), i.e., solving the equation \( x^2 = 1.9^x \). Consider \( f \) on the interval \([2, 6]\):

\[
f(2) = 4 - 1.9^2 = 0.39 > 0 \quad \text{and} \quad f(6) = 36 - 1.9^6 = -11.045881 < 0.
\]

By the IMVT there is at least one number \( c \) in \([2, 6]\) satisfying \( f(c) = 0 \), i.e., solving the equation \( x^2 = 1.9^x \).

This completes the problem.
8.a) Let \( f(x) = 5x^3 - x + 2 \) on \([-2, 0]\). Then \( f \) is continuous (since it's a polynomial) and \( m = 0 \) is between \( f(-2) = -31 \) and \( f(0) = 2 \). So by the IMUT there is a number \( c \) satisfying \( f(c) = 0 \), i.e., \( 5c^3 - c + 2 = 0 \), where \(-2 \leq c \leq 0\).

b) Since \( n \) is odd and \( a_n > 0 \), \( \lim_{x \to +\infty} P(x) = +\infty \) and \( \lim_{x \to -\infty} P(x) = -\infty \). Thus, there are numbers \( a \) and \( b \) satisfying \( P(a) > 0 \) and \( P(b) < 0 \). Since \( P \) is continuous (since it's a polynomial) and \( m = 0 \) is between \( P(a) \) and \( P(b) \), it follows from the IMUT that there is a number \( r \) between \( a \) and \( b \) so that \( P(r) = 0 \).
9.) Let \( f(x) = \frac{1}{x+3} - e^x \), which is continuous on the interval \([1, 0]\). Constant \( m = 0 \) is between \( f(1) = \frac{1}{2} - \frac{1}{e} > 0 \) and \( f(0) = -\frac{2}{3} \). Thus, by the IMUT there is at least one number \( c \), \(-1 \leq c \leq 0\), satisfying \( f(c) = 0 \), i.e., \( \frac{1}{c+3} - e^c = 0 \).

10.) Let \( f(x) = x^3 - 2^x \), which is continuous on the interval \([0, 2]\). Constant \( m = 0 \) is between \( f(0) = -1 \) and \( f(2) = 4 \). Thus, by the IMUT there is at least one number \( c \), \(0 \leq c \leq 2\), satisfying \( f(c) = 0 \), i.e., \( c^3 - 2^c = 0 \) or \( c^3 = 2^c \).

11.) Let \( h(x) = f(x) - g(x) \) on the interval \([a, b]\), which is continuous since \( f \) and \( g \) are continuous. Let \( m = 0 \) and note that \( h(a) = f(a) - g(a) < 0 \) and \( h(b) = f(b) - g(b) > 0 \). Since \( m \) is between \( h(a) \) and \( h(b) \), the IMUT guarantees that there is a number \( c \), \( a \leq c \leq b \), satisfying \( h(c) = 0 \), i.e., \( f(c) - g(c) = 0 \), i.e., \( f(c) = g(c) \).
12) \( h(x) = \frac{f(x)}{g(x)} \) is continuous on \([a, b]\) since \( f \) and \( g \) are continuous on \([a, b]\) and \( g(x) > 0 \) on \([a, b]\); 
\[ h(a) = \frac{f(a)}{g(a)} = 1 \quad \text{and} \quad h(b) = \frac{f(b)}{g(b)} < 0 \] 
so by I M U T there is \( c, \ a \leq c \leq b \) satisfying \( h(c) = \frac{1}{2} \), i.e., \( \frac{f(c)}{g(c)} = \frac{1}{2} \), i.e., \( 2f(c) = g(c) \). 

13) \[ 1, 5, 9, 17, \ldots, 12045 \] is generated 
by the formula \( 4n - 3 \) for \( n \geq 1 \). 
\[ 4n - 3 = 12045 \implies 4n = 12048 \implies n = 3012 \] numbers are in the list. 

The total sum of these numbers is 
\[ \sum_{n=1}^{3012} (4n - 3) = 4 \sum_{n=1}^{3012} n - 3 \sum_{n=1}^{3012} 1 \]
\[ = 4 \frac{(3012)(3012 + 1)}{2} - 3(3012) \]
\[ = 18,141,276. \]