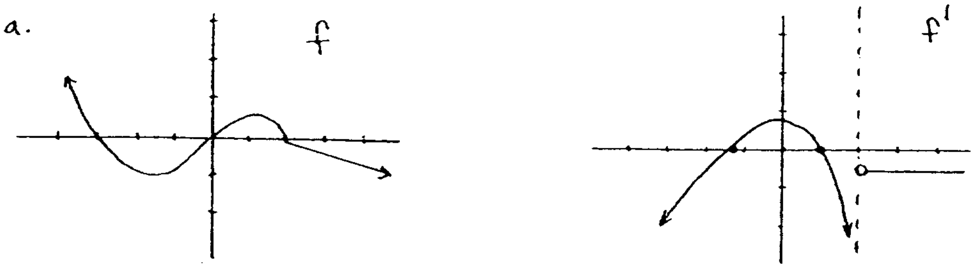
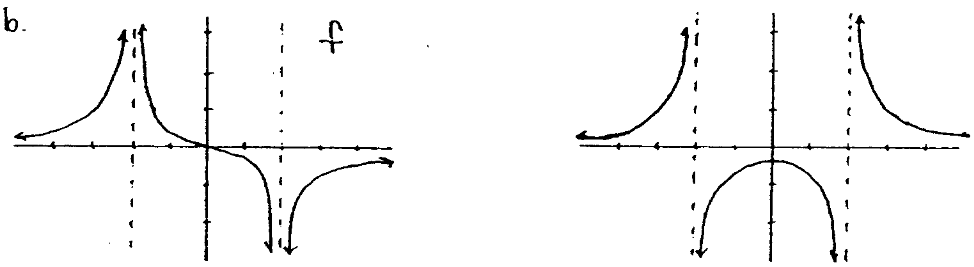
ESP
Kouba
Worksheet 9 Solutions

1. Sketch the graph of f ' by using the graph of f .

a.

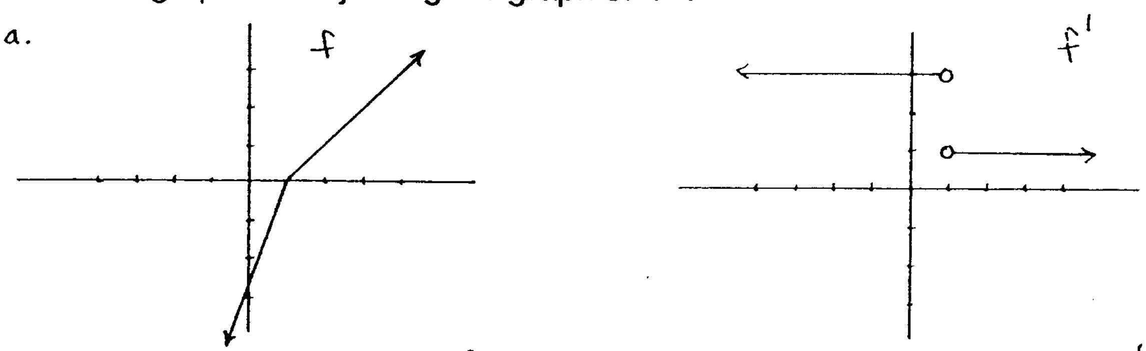


b.

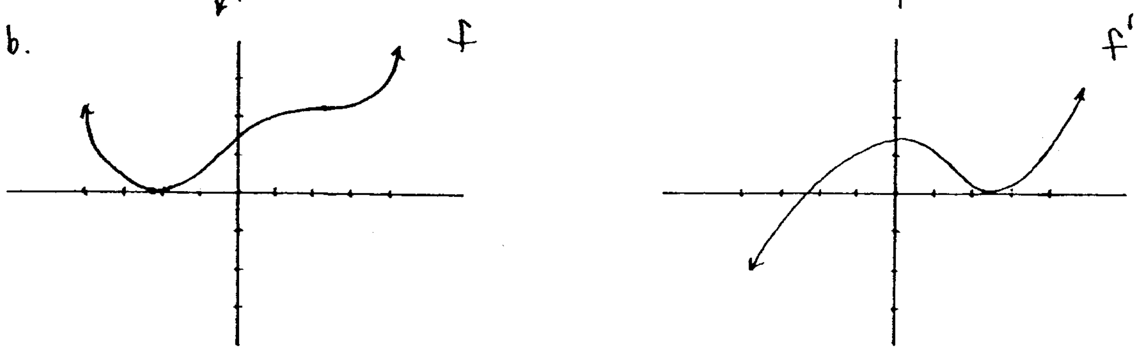


2. Sketch a graph of f by using the graph of f ' .

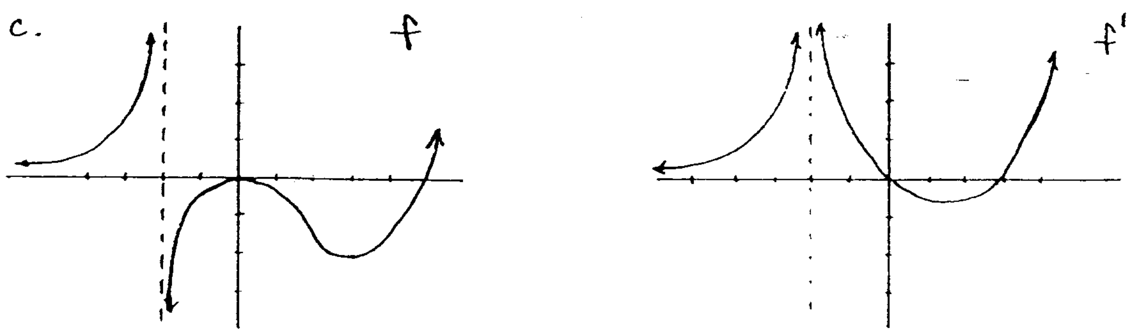
a.



b.



c.



3.) a.) Let $g(x) = x^{1/3}$ and $h(x) = \sin x + 17$ then

$f(x) = g(h(x)) = g(\sin x + 17) = (\sin x + 17)^{1/3}$.

b.) Let $g(x) = 1 + \sqrt{x}$ and $h(x) = x^3 + \tan x$ then

$f(x) = g(h(x)) = g(x^3 + \tan x) = 1 + \sqrt{x^3 + \tan x}$.

c.) Let $g(x) = \frac{5}{2}\cos x$ and $h(x) = \pi \sec x$ then

$f(x) = g(h(x)) = g(\pi \sec x) = \frac{5}{2}\cos(\pi \sec x)$.

4.) a.) Let $g(x) = 7x^3 - 4$, $h(x) = 5x^2 + 2$, and $s(x) = 3x + 1$

then $f(x) = g(h(s(x))) = g(h(3x+1))$

$= g(5(3x+1)^2 + 2) = 7(5(3x+1)^2 + 2)^3 - 4$.

b.) Let $g(x) = \tan x$, $h(x) = \sqrt{x}$, and $s(x) = x^3 + x - 7$ then

$f(x) = g(h(s(x))) = g(h(x^3 + x - 7))$

$= g(\sqrt{x^3 + x - 7}) = \tan\sqrt{x^3 + x - 7}$.

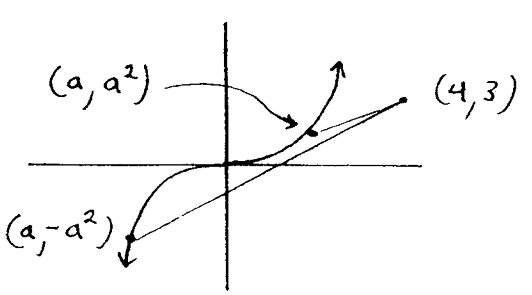
c.) Let $g(x) = x^{50}$, $h(x) = x + \tan^2(x+1)$, and

$s(x) = x^5$ then $f(x) = g(h(s(x)))$

$= g(h(x^5)) = g(x^5 + \tan^2(x^5+1))$

$= (x^5 + \tan^2(x^5+1))^{50}$

5.)



For $x > 0$, $y' = 2x$

so slope of tangent line at (a, a^2) is

$$2a = \frac{3-a^2}{4-a} \Rightarrow 8a - 2a^2 = 3 - a^2 \Rightarrow$$

$$0 = a^2 - 8a + 3 \Rightarrow a = \frac{8 \pm \sqrt{64-12}}{2} = 4 \pm \sqrt{13};$$

<u>For $x<0$</u>, $y' = -2x$ so slope of tangent line at $(a, -a^2)$ is

$$-2a = \frac{3-(-a^2)}{4-a} \Rightarrow -8a + 2a^2 = 3 + a^2 \Rightarrow$$

$$a^2 - 8a - 3 = 0 \Rightarrow a = \frac{8 \pm \sqrt{64+12}}{2} = 4 \pm \sqrt{19},$$

but $4+\sqrt{19} > 0$ so use $a = 4 - \sqrt{19}$.

6.) a.) $y' = (3x) \cdot \sec^2 x + (3) \cdot \tan x$

b.) $y' = \sec^2 (3x)^2 \cdot 2(3x) \cdot 3$

c.) $y' = 2\tan(3x) \cdot \sec^2(3x) \cdot 3$

d.) $y' = 2\tan(3x) \cdot \sec^2(3x) \cdot 3$

e.) $f'(x) = \sin(2x) \cdot (0) + 2\cos(2x) \cdot \cos(\pi^3)$

f.) $g'(x) = 3\cos^2(\pi x) \cdot {}^-\sin(\pi x) \cdot \pi$

g.) $y' = \cos(\cos(3x)) \cdot {}^-\sin(3x) \cdot 3$

h.) $g'(x) = \sqrt{x} \cdot \sec x \tan x + \frac{1}{2} x^{-1/2} \cdot \sec x$

i.) $f'(x) = \cos\sqrt{x} \cdot \frac{1}{2} x^{-1/2}$

j.) $y' = \dfrac{(1+\cos x)\cdot \cos x - \sin x \cdot (-\sin x)}{(1+\cos x)^2}$

k.) $y' = -70\left(x+(9x-1)^4\right)^{69} \cdot \{1 + 4(9x-1)^3 \cdot 9\}$

l.) $y' = \frac{1}{2}\left(1+\sqrt{1+\sqrt{1+x}}\right)^{-1/2} \cdot \frac{1}{2}\left(1+\sqrt{1+x}\right)^{-1/2} \cdot \frac{1}{2}(1+x)^{-1/2} \cdot 1$

m.) $f'(x) = 10\left(\dfrac{x}{x+7}\right)^9 \cdot \dfrac{(x+7)(1) - x(1)}{(x+7)^2}$

n.) $f'(x) = \dfrac{(x-7)^{1/5} \cdot 3(1-x)^2 \cdot (-1) - (1-x)^3 \cdot \frac{1}{5}(x-7)^{-4/5}}{(x-7)^{2/5}}$

o.) $g'(x) = 0$!

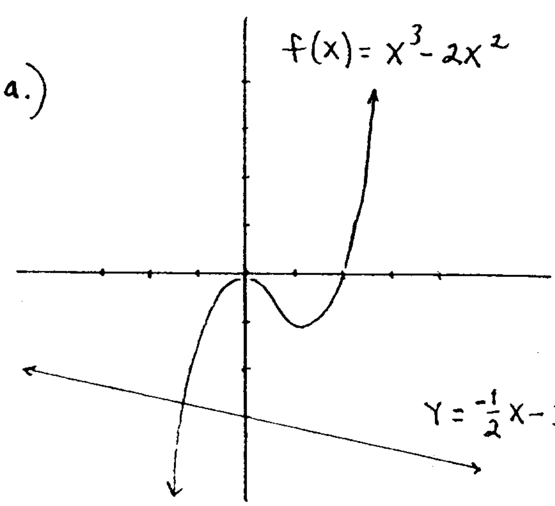
7.) $f'(x) = 1\cdot(x-1)^5(2+x)^3 + x\cdot 5(x-1)^4(2+x)^3 + x(x-1)^5 \cdot 3(2+x)^2$

$= (x-1)^4(2+x)^2 \cdot \{(x-1)(2+x) + 5x(2+x) + 3x(x-1)\}$

$= (x-1)^4(2+x)^2 \cdot \{9x^2 + 8x - 2\} = 0 \Rightarrow$

$x=1$, $x=-2$, and $x = \dfrac{-8 \pm \sqrt{64+72}}{18} = \dfrac{-4 \pm \sqrt{34}}{9}$.

8.) a.)



b.) Find all points on the graph of f whose slopes of tangent line are

$m = \dfrac{-1}{\left(\frac{-1}{2}\right)} = 2$:

$f'(x) = 3x^2 - 4x = 2 \Rightarrow 3x^2 - 4x - 2 = 0 \Rightarrow$

$$x = \frac{4 \pm \sqrt{16+24}}{6} = \frac{2 \pm \sqrt{10}}{3} \quad ;$$

if $x = \dfrac{2+\sqrt{10}}{3}$ then $y = \left(\dfrac{2+\sqrt{10}}{3}\right)^3 - 2\left(\dfrac{2+\sqrt{10}}{3}\right)^2$

$= \frac{1}{27}(68 + 22\sqrt{10}) - \frac{2}{9}(14 + 4\sqrt{10}) = \frac{-16}{27} - \frac{2}{27}\sqrt{10}$ so

orthogonal line is : $\boxed{y - \left(\frac{-16}{27} - \frac{2}{27}\sqrt{10}\right) = \frac{-1}{2}\left(x - \frac{2+\sqrt{10}}{3}\right)}$;

if $x = \dfrac{2-\sqrt{10}}{3}$ then $y = \left(\dfrac{2-\sqrt{10}}{3}\right)^3 - 2\left(\dfrac{2-\sqrt{10}}{3}\right)^2$

$= \frac{1}{27}(68 - 22\sqrt{10}) - \frac{2}{9}(14 - 4\sqrt{10}) = \frac{-16}{27} + \frac{2}{27}\sqrt{10}$ so

orthogonal line is : $\boxed{y - \left(\frac{-16}{27} + \frac{2}{27}\sqrt{10}\right) = \frac{-1}{2}\left(x - \frac{2-\sqrt{10}}{3}\right)}$.