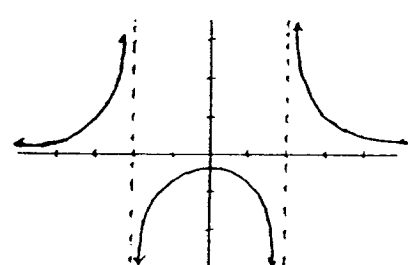
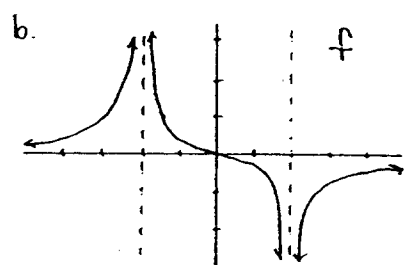
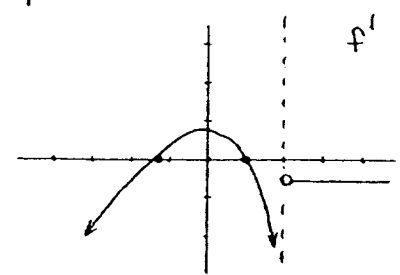
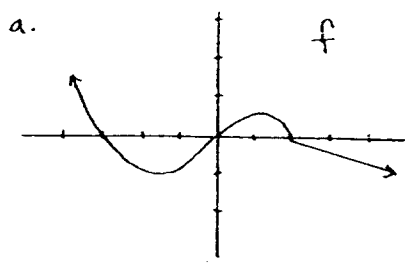
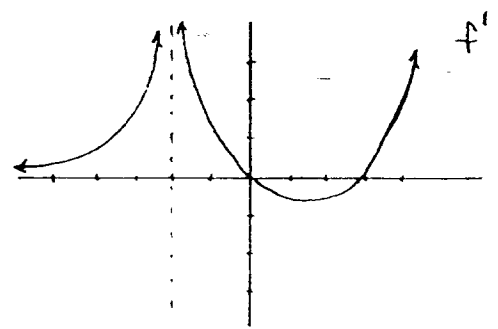
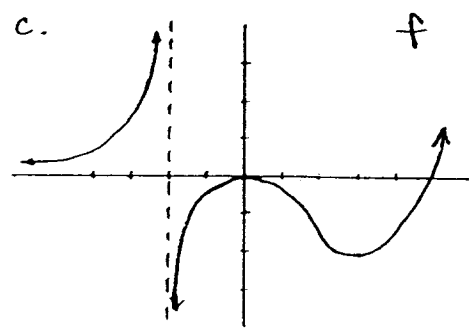
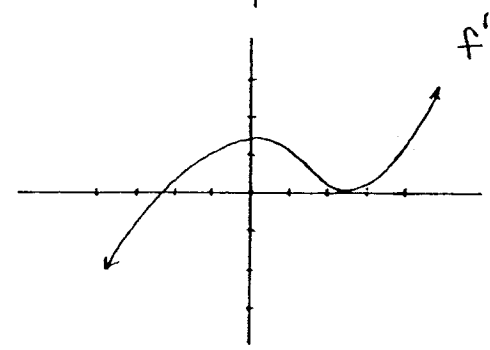
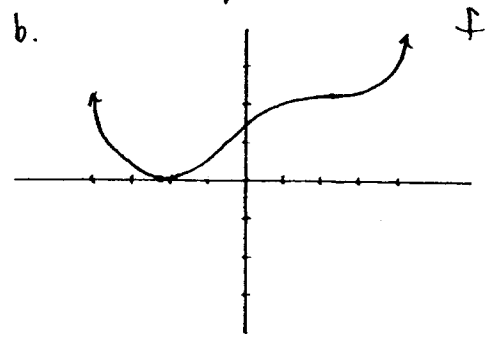
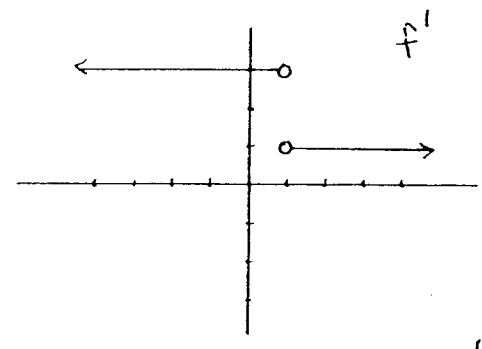
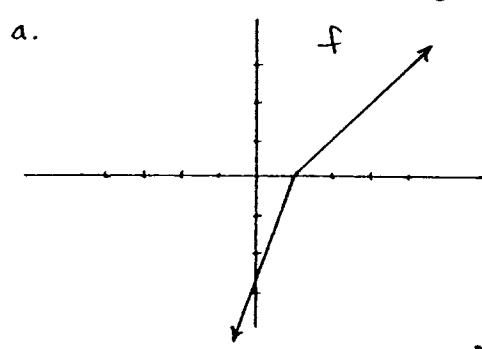


ESP
Kouba
Worksheet 9 Solutions

1. Sketch the graph of f' by using the graph of f .



2. Sketch a graph of f by using the graph of f' .



3.) a.) Let $g(x) = x^{1/3}$ and $h(x) = \sin x + 17$ then

$$f(x) = g(h(x)) = g(\sin x + 17) = (\sin x + 17)^{1/3}$$

b.) Let $g(x) = 1 + \sqrt{x}$ and $h(x) = x^3 + \tan x$ then

$$f(x) = g(h(x)) = g(x^3 + \tan x) = 1 + \sqrt{x^3 + \tan x}$$

c.) Let $g(x) = \frac{5}{2} \cos x$ and $h(x) = \pi \sec x$ then

$$f(x) = g(h(x)) = g(\pi \sec x) = \frac{5}{2} \cos(\pi \sec x)$$

4.) a.) Let $g(x) = 7x^3 - 4$, $h(x) = 5x^2 + 2$, and $s(x) = 3x + 1$

$$\text{then } f(x) = g(h(s(x))) = g(h(3x + 1))$$

$$= g(5(3x + 1)^2 + 2) = 7(5(3x + 1)^2 + 2)^3 - 4$$

b.) Let $g(x) = \tan x$, $h(x) = \sqrt{x}$, and $s(x) = x^3 + x - 7$ then

$$f(x) = g(h(s(x))) = g(h(x^3 + x - 7))$$

$$= g(\sqrt{x^3 + x - 7}) = \tan \sqrt{x^3 + x - 7}$$

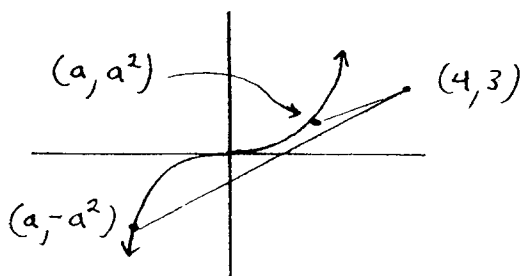
c.) Let $g(x) = x^{50}$, $h(x) = x + \tan^2(x + 1)$, and

$$s(x) = x^5 \text{ then } f(x) = g(h(s(x)))$$

$$= g(h(x^5)) = g(x^5 + \tan^2(x^5 + 1))$$

$$= (x^5 + \tan^2(x^5 + 1))^{50}$$

5.)



For $x > 0$, $y' = 2x$

so slope of tangent line at (a, a^2) is

$$2a = \frac{3-a^2}{4-a} \Rightarrow 8a - 2a^2 = 3 - a^2 \Rightarrow$$

$$0 = a^2 - 8a + 3 \Rightarrow a = \frac{8 \pm \sqrt{64-12}}{2} = \boxed{4 \pm \sqrt{13}} ;$$

For $x < 0$, $y' = -2x$ so slope of tangent line at $(a, -a^2)$ is

$$-2a = \frac{3 - (-a^2)}{4 - a} \Rightarrow -8a + 2a^2 = 3 + a^2 \Rightarrow$$

$$a^2 - 8a - 3 = 0 \Rightarrow a = \frac{8 \pm \sqrt{64+12}}{2} = 4 \pm \sqrt{19} ,$$

but $4 + \sqrt{19} > 0$ so use $a = \boxed{4 - \sqrt{19}}$.

6.) a) $y' = (3x) \cdot \sec^2 x + (3) \cdot \tan x$

b) $y' = \sec^2 (3x)^2 \cdot 2(3x) \cdot 3$

c) $y' = 2 \tan(3x) \cdot \sec^2(3x) \cdot 3$

d) $y' = 2 \tan(3x) \cdot \sec^2(3x) \cdot 3$

e) $f'(x) = \sin(2x) \cdot (0) + 2 \cos(2x) \cdot \cos(\pi^3)$

f) $g'(x) = 3 \cos^2(\pi x) \cdot -\sin(\pi x) \cdot \pi$

g) $y' = \cos(\cos(3x)) \cdot -\sin(3x) \cdot 3$

h) $g'(x) = \sqrt{x} \cdot \sec x \tan x + \frac{1}{2} x^{-1/2} \cdot \sec x$

i) $f'(x) = \cos \sqrt{x} \cdot \frac{1}{2} x^{-1/2}$

$$j.) \quad y' = \frac{(1 + \cos x) \cdot \cos x - \sin x \cdot (-\sin x)}{(1 + \cos x)^2}$$

$$k.) \quad y' = -70(x + (9x-1)^4)^{69} \cdot \{1 + 4(9x-1)^3 \cdot 9\}$$

$$l.) \quad y' = \frac{1}{2}(1 + \sqrt{1+x})^{-\frac{1}{2}} \cdot \frac{1}{2}(1 + \sqrt{1+x})^{-\frac{1}{2}} \cdot \frac{1}{2}(1+x)^{-\frac{1}{2}} \cdot 1$$

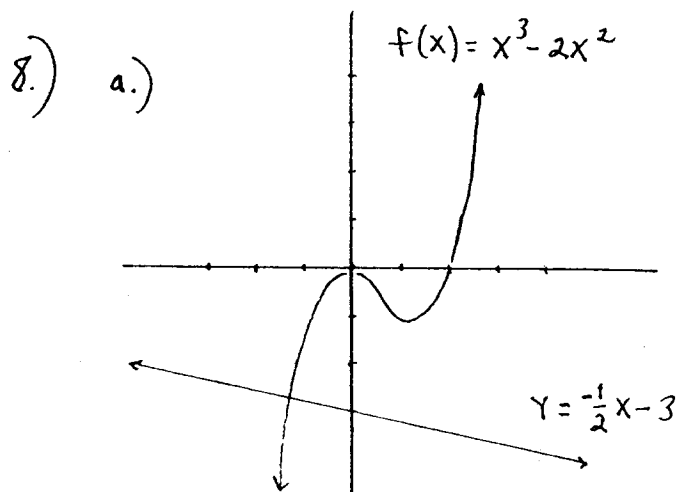
$$m.) \quad f'(x) = 10 \left(\frac{x}{x+7} \right)^9 \cdot \frac{(x+7)(1) - x(1)}{(x+7)^2}$$

$$n.) \quad f'(x) = \frac{(x-7)^{\frac{1}{5}} \cdot 3(1-x)^2 \cdot (-1) - (1-x)^3 \cdot \frac{1}{5}(x-7)^{-\frac{4}{5}}}{(x-7)^{\frac{2}{5}}}$$

$$o.) \quad g'(x) = 0 !$$

$$\begin{aligned} 7.) \quad f'(x) &= 1 \cdot (x-1)^5 \cdot (2+x)^3 + x \cdot 5(x-1)^4 \cdot (2+x)^3 + x(x-1)^5 \cdot 3(2+x)^2 \\ &= (x-1)^4 (2+x)^2 \cdot \{ (x-1)(2+x) + 5x(2+x) + 3x(x-1) \} \\ &= (x-1)^4 (2+x)^2 \cdot \{ 9x^2 + 8x - 2 \} = 0 \Rightarrow \end{aligned}$$

$$\boxed{x=1}, \boxed{x=-2}, \text{ and } x = \frac{-8 \pm \sqrt{64+72}}{18} = \boxed{\frac{-4 \pm \sqrt{34}}{9}}$$



b.) Find all points on the graph of f whose slopes of tangent line are

$$m = \frac{-1}{\left(\frac{-1}{2}\right)} = 2 :$$

$$f'(x) = 3x^2 - 4x = 2 \Rightarrow 3x^2 - 4x - 2 = 0 \Rightarrow$$

$$x = \frac{4 \pm \sqrt{16 + 24}}{6} = \frac{2 \pm \sqrt{10}}{3} ;$$

$$\underline{\text{if } x = \frac{2 + \sqrt{10}}{3}} \quad \text{then } y = \left(\frac{2 + \sqrt{10}}{3}\right)^3 - 2\left(\frac{2 + \sqrt{10}}{3}\right)^2$$

$$= \frac{1}{27}(68 + 22\sqrt{10}) - \frac{2}{9}(14 + 4\sqrt{10}) = \frac{-16}{27} - \frac{2}{27}\sqrt{10} \quad \text{so}$$

$$\text{orthogonal line is: } \boxed{y - \left(\frac{-16}{27} - \frac{2}{27}\sqrt{10}\right) = \frac{-1}{2} \left(x - \frac{2 + \sqrt{10}}{3}\right)} ;$$

$$\underline{\text{if } x = \frac{2 - \sqrt{10}}{3}} \quad \text{then } y = \left(\frac{2 - \sqrt{10}}{3}\right)^3 - 2\left(\frac{2 - \sqrt{10}}{3}\right)^2$$

$$= \frac{1}{27}(68 - 22\sqrt{10}) - \frac{2}{9}(14 - 4\sqrt{10}) = \frac{-16}{27} + \frac{2}{27}\sqrt{10} \quad \text{so}$$

$$\text{orthogonal line is: } \boxed{y - \left(\frac{-16}{27} + \frac{2}{27}\sqrt{10}\right) = \frac{-1}{2} \left(x - \frac{2 - \sqrt{10}}{3}\right)} .$$