Math 21B
Kouba
Discussion Sheet 10

1.) Compute the following improper integrals.

a.) $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^2}\,dx$ b.) $\displaystyle\int_0^5 \frac{1}{\sqrt{25-x^2}}\,dx$ c.) $\displaystyle\int_1^\infty \frac{24}{2x^2+5x+2}\,dx$

d.) $\displaystyle\int_0^5 \frac{8x}{x^2-9}\,dx$ e.) $\displaystyle\int_{-\infty}^\infty x^2 e^{x^3}\,dx$ f.) $\displaystyle\int_0^{\pi/2} \csc x \cot x\,dx$

g.) $\displaystyle\int_0^\infty xe^{-5x}\,dx$ h.) $\displaystyle\int_0^\infty \frac{1}{x^2}\,dx$ i.) $\displaystyle\int_0^1 \ln x\,dx$

j.) $\displaystyle\int_0^e x\ln x\,dx$ k.) $\displaystyle\int_0^3 \frac{e^{2x}}{e^{2x}-5}\,dx$ l.) $\displaystyle\int_0^\infty \frac{e^{-1/x}}{x^2}\,dx$

2.) Use the Comparison Test or Absolute Convergence Test to show that each of the following improper integrals converges, i.e., is finite.

a.) $\displaystyle\int_1^\infty \frac{1}{\sqrt{x^3+16}}\,dx$ b.) $\displaystyle\int_3^\infty \frac{\cos x-\sin 2x}{x+x^2}\,dx$

3.) Use Pappus Theorem (See p. 498, problem 9.) to find the centroid $(\bar{x},\bar{y})$ of the triangle with vertices $(0,0),(3,0)$, and $(0,4)$. HINT : The volume of a cone of radius r and height h is $V=\frac{1}{3}\pi r^2 h$.

4.) Find the area of the region bounded by the graphs of $\theta=\pi/6$, $\theta=\pi/4$, and $r=\sec\theta$.

5.) Find the area of the region lying inside the graph of $r=2+\sin\theta$ and outside the graph of $r=\cos\theta$.

6.) Find the area of the region lying inside the graph of $r=4\sin\theta$ and above the line $r=\csc\theta$.

7.) Compute the arc length of the given curve on the indicated interval.

a.) $y=x^{5/4}$ on the interval $[0,1]$

b.) $x=\cos t+t\sin t$ and $y=\sin t-t\cos t$ on the interval $[\pi/6,\pi/4]$

b.) $r=\sin^2(\theta/2)$ on the interval $[0,\pi]$

8.) Find the maximum y-value and the maximum x-value on the graph of $r=1-\sin\theta$.