

Math 21B  
Kouba  
Discussion Sheet 2

1.) Determine the following indefinite integrals (antiderivatives).

a.)  $\int \frac{x^2 + 1}{x^3} dx$     b.)  $\int \frac{x^2 + 1}{x + 3} dx$     c.)  $\int \frac{x^2}{x^3 + 1} dx$

2.) Evaluate the following definite integrals.

a.)  $\int_0^{\frac{\pi}{2}} \cos x e^{\sin x} dx$     b.)  $\int_{-1}^1 5^x dx$     c.)  $\int_0^{\pi} 5 \sec^2 3x dx$

3.) Set up a definite integral and compute the volume of a solid located between a plane at  $x = 1$  and a plane located at  $x = 5$  if the cross-sectional area of the solid by a plane perpendicular to the  $x$ -axis at  $x$  is  $6x^3$  square centimeters.

4.) Evaluate the following. HINT: Use problem 34 on page 265.

a.)  $\sum_{i=0}^{20} 3^i$     b.)  $\sum_{i=2}^{15} 2^{2i}$     c.)  $\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{i=1}^n 2^{\frac{i}{n}}$

5.) Differentiate each :    a.)  $F(x) = \int_{-100}^{\sin 3x} \sqrt{1+t^2} dt$     b.)  $F(x) = \int_{\tan x}^{\arctan x} 5^{t^2} dt$

6.) Find the average value of each of the following functions over the given interval. Draw a sketch showing the connection between your answer and the definite integral.

a.)  $f(x) = x^3 + 1$  on  $[-1, 1]$     b.)  $f(x) = 5 + \sqrt{x}$  on  $[0, 4]$

7.) If  $\int_{-2}^1 f(x) dx = 3$  and  $\int_{-2}^3 f(x) dx = -2$ . What is the value of  $\int_3^1 f(x) dx$  ?

8.) A thin rod is 16 cm. long. Its density  $x$  cm. from its left end is given by  $2 + \sqrt{x}$  gm./cm.

a.) Estimate the rod's total mass by using four equal subdivisions and midpoints to estimate densities.

b.) Set up a definite integral and compute the exact mass of the rod.

9.) Consider the following sequence of numbers : 1, 5, 9, 13, 17, 21, 25, ... Find the sum of the 61st through the 127th numbers in this sequence.

10.) Use the limit definition of a definite integral to evaluate

a.)  $\int_0^1 x^3 dx$     b.)  $\int_0^2 e^x dx$

THE FOLLOWING PROBLEM IS FOR RECREATIONAL PURPOSES ONLY.

11.) Connect 6 toothpicks end-to-end to form 4 triangles.