

Math 21B
 Kouba
 Challenge Sheet 3

1.) Assume that the temperature of the sidewalk on a hot summer day is given by $T(t) = 100 + 30e^{t-4}$ degrees Fahrenheit at hour t for $0 \leq t \leq 4$.

- a.) What is the sidewalk's temperature when $t = 0$ hours ?
- b.) What is the sidewalk's temperature when $t = 4$ hours ?
- c.) What is the sidewalk's average temperature for $0 \leq t \leq 4$?

2.) Convert the following limit to a definite integral and then evaluate the integral :

$$\lim_{n \rightarrow \infty} \sum_{i=1}^{2n} \left(\frac{3i}{n^2} - \frac{1}{n^2} \right)^2 n$$

3.) The average value of function $f(x) = 3x^2 + 4x$ on the interval $[0, b]$ is 35. What is the value of b ?

4.) Consider the function $f(t) = \begin{cases} 1, & \text{if } 0 \leq t < 1 \\ 2, & \text{if } 1 \leq t \leq 2 \end{cases}$ and let $G(x) = \int_0^x f(t) dt$.

- i.) Sketch the graph of f on the interval $0 \leq t \leq 2$.
 - ii.) Is f continuous on the interval $0 \leq t \leq 2$?
 - iii.) What is the average value of f on the interval $0 \leq t \leq 2$?
- i.) Determine a "formula" for G .
 - ii.) Sketch the graph of G on the interval $0 \leq x \leq 2$.
 - iii.) Is G continuous on the interval $0 \leq x \leq 2$?
 - iv.) What is the average value of G on the interval $0 \leq x \leq 2$?

5.) Determine all possible functions f which satisfy the equation

- a.) $f'(t) = 2t(1 + t^2)^5$.
- b.) $f'(t) = 2t(1 + (f(t))^2)$.

6.) Let $C(t)$ be the number of crocodiles in a habitat at time t years. Initially there are 50 crocodiles and after 5 years there are 75 crocodiles. If the rate at which the number of crocodiles changes at time t is directly proportional to the square of the number of crocodiles at time t , how many crocodiles will there be when $t = 10$ years ? $t = 14$ years ? $t = 14.999$ years ? What is your conclusion about this mathematical model ?

7.) Determine the radius of the inscribed circle.

