

Math 21B
 Kouba
 Challenge Sheet 5

1.) Use any method to integrate the following.

a.) $\int \frac{1}{x+1} dx$

b.) $\int \frac{1}{x^2+1} dx$

c.) $\int \frac{1}{x^3+1} dx$

d.) $\int \frac{1}{x^4+1} dx$

e.) $\int \frac{1}{x(x^4+1)} dx$

f.) $\int \frac{x}{x^4+1} dx$

g.) $\int \frac{\cos x}{1-\sin x} dx$

h.) $\int \frac{1}{1-\sin x} dx$

i.) $\int \frac{1}{\cos x - \sin x} dx$

j.) $\int \frac{\cos x}{\sin x - \sin^2 x} dx$

k.) $\int \frac{1}{x^2+8x+16} dx$

l.) $\int \frac{x+4}{x^2+8x+18} dx$

m.) $\int \frac{x}{x^2+8x+20} dx$

n.) $\int \frac{x+1}{3x^2+8x+20} dx$

2.) a.) Let f be a continuous function on the interval $[0, a]$. Show that

$$\int_0^a f(x) dx = \int_0^a f(a-x) dx$$

b.) Use part a.) to evaluate $\int_0^{\pi/2} \frac{\sin^3 x}{\sin^3 x + \cos^3 x} dx$

3.) Inside the larger semi-circle of radius 4 is inscribed a square. Inside the square sit a smaller semi-circle and an isosceles triangle (with base parallel to the diameter of the larger semi-circle). Determine the area of the triangle.

