

Math 21B
 Kouba
 Challenge Sheet 6

1.) SET UP the partial fractions decomposition for each rational function. DO NOT SOLVE for the unknown constants.

a.) $\frac{x^3 + x^2 + x + 3}{x^4 - 1}$

b.) $\frac{700}{x^3(x^2 + 5)^3}$

c.) $\frac{x^5 + x - 100}{x^8 - 1}$

2.) Use any method to integrate the following.

a.) $\int \frac{1}{x(x^2 + 1)} dx$

b.) $\int \frac{1}{x(x^9 + 1)} dx$

c.) $\int \frac{1}{x^{1/2} - x^{3/2}} dx$

d.) $\int (x - 1)\sqrt{x - 2} dx$

e.) $\int \sqrt{x - 1} \sqrt{x - 2} dx$

f.) $\int \frac{\sqrt{x - 1}}{\sqrt{x - 2}} dx$

g.) $\int x \cdot \sqrt{4 - x^2} dx$

h.) $\int \sqrt{x^2 - 4} dx$

i.) $\int \sqrt{4 - x^2} dx$

j.) $\int \sqrt{x^2 + 4} dx$

k.) $\int x \cdot \sqrt{16 - x^4} dx$

l.) $\int x^7 \cdot \sqrt{16 - x^4} dx$

3.) Derive recursion formulas for each integral (n is a positive integer and b is a constant).

a.) $\int x^n \sin bx dx$

b.) $\int \cot^n x dx$

4.) You are standing with an empty pail at point A, 3 miles from a road and 5 miles from a river. You must walk to the river, fill the pail with water, return the pail to the road, then walk back to point B on the river. Determine the shortest possible walking distance required to complete this task.

