

Math 21B
 Kouba
 Challenge Sheet 9

1.) Evaluate the following improper integrals. Make careful use of limits and notation.

a.) $\int_0^1 \frac{1}{\sqrt{x}} dx$

b.) $\int_1^\infty \frac{1}{\sqrt{x}} dx$

c.) $\int_{\sqrt{3}}^\infty \frac{1}{x^2 + 1} dx$

d.) $\int_0^\infty \frac{10x^4 + 6x^2 + 2}{x^5 + x^3 + x + 7} dx$

e.) $\int_4^\infty \frac{1}{x^2 - 9} dx$

f.) $\int_{-1}^1 \frac{1}{\sqrt{|x|}} dx$

g.) $\int_0^1 \ln x dx$ (HINT: You will need integration by parts and L'Hopital's Rule.)

2.) Use the following tests to determine if the following improper integrals converge or diverge— Comparison Test for Convergence, Comparison Test for Divergence, or Absolute Convergence Test.

a.) $\int_1^\infty \frac{1}{\sqrt{x}\sqrt{x+2}\sqrt{x+4}} dx$

b.) $\int_0^1 \frac{1}{\sqrt{x}\sqrt{x+2}\sqrt{x+4}} dx$

c.) $\int_0^1 \frac{\ln x}{1-x^2} dx$ (HINT: Let $f(x) = \frac{\ln x}{1-x^2}$ and consider $\lim_{x \rightarrow 0^+} f(x)$, $\lim_{x \rightarrow 1^-} f(x)$, and the graph of f .)

d.) $\int_1^\infty \frac{\sin x}{x^2} dx$

e.) $\int_0^1 \frac{\cos x}{x} dx$ (HINT: Consider the maximum and minimum values of $y = \frac{\cos x}{x}$ on the interval $[0, 1]$.)

f.) $\int_1^\infty \frac{\cos x}{x} dx$

g.) $\int_0^1 \frac{\sin x}{x^2} dx$ (HINT: Recall that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.)

h.) $\int_0^\infty \frac{\sin x}{x} dx$

i.) $\int_0^\infty \cos(e^x) dx$ (HINT: Begin with a u-substitution.)

3.) Consider the region above the x-axis and between the two given circles. Set up integrals for the centroid, (\bar{x}, \bar{y}) , of the region. Use a graphing calculator to evaluate the integrals. Does the centroid lie inside the region?

