ESP
Kouba
Worksheet 11 Solutions

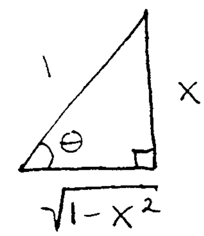
1.) a.) Let $x = \sin\theta \rightarrow dx = \cos\theta \, d\theta \rightarrow$

$$\int \sqrt{1-x^2}\, dx = \int \sqrt{1-\sin^2\theta} \cdot \cos\theta \, d\theta = \int \cos^2\theta \, d\theta$$

$$= \int \frac{1+\cos 2\theta}{2}\, d\theta = \frac{1}{2}\theta + \frac{1}{4}\sin 2\theta + c$$

$$= \frac{1}{2}\theta + \frac{1}{4}\cdot 2\sin\theta\cos\theta + c$$

$$= \frac{1}{2}\arcsin x + \frac{1}{2}\cdot x \cdot \sqrt{1-x^2} + c$$

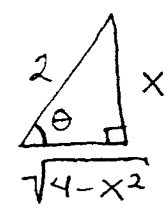


b.) Let $x = 2\sin\theta \rightarrow dx = 2\cos\theta\, d\theta \rightarrow$

$$\int \sqrt{4-x^2}\, dx = \int \sqrt{4-4\sin^2\theta} \cdot 2\cos\theta\, d\theta = 4\int \cos^2\theta\, d\theta$$

$$= \cdots = 2\theta + 2\sin\theta\cos\theta + c$$

$$= 2\arcsin\left(\frac{x}{2}\right) + 2\cdot\frac{x}{2}\cdot\frac{\sqrt{4-x^2}}{2} + c$$



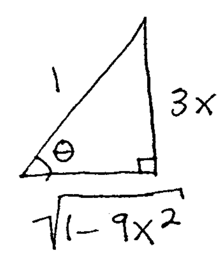
c.) Let $x = \frac{1}{3}\sin\theta \rightarrow dx = \frac{1}{3}\cos\theta\, d\theta \rightarrow$

$$\int \sqrt{1-9x^2}\, dx = \int \sqrt{1-\cancel{9}\left(\frac{1}{\cancel{9}}\sin^2\theta\right)} \cdot \frac{1}{3}\cos\theta\, d\theta$$

$$= \frac{1}{3}\int \cos^2\theta\, d\theta = \cdots = \frac{1}{6}\theta + \frac{1}{6}\sin\theta\cos\theta + c$$

$$= \frac{1}{6}\arcsin(3x)$$

$$+ \frac{1}{6}(3x)\cdot\sqrt{1-9x^2} + c$$



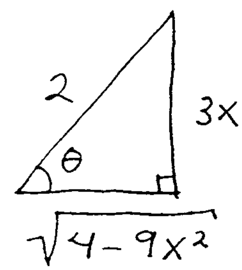
d.) Let $x = \frac{2}{3}\sin\theta \rightarrow dx = \frac{2}{3}\cos\theta \, d\theta \rightarrow$

$$\int \sqrt{4-9x^2}\, dx = \int \sqrt{4 - \cancel{9}\left(\frac{4}{\cancel{9}}\sin^2\theta\right)} \cdot \frac{2}{3}\cos\theta \, d\theta$$

$$= \frac{4}{3}\int \cos^2\theta \, d\theta = \cdots = \frac{2}{3}\theta + \frac{2}{3}\sin\theta\cos\theta + C$$

$$= \frac{2}{3}\arcsin\left(\frac{3x}{2}\right)$$

$$+ \frac{2}{3} \cdot \frac{3x}{2} \cdot \frac{\sqrt{4-9x^2}}{2} + C$$



e.) Let $x = \sec\theta \rightarrow dx = \sec\theta\tan\theta \, d\theta \rightarrow$

$$\int \sqrt{x^2-1}\, dx = \int \sqrt{\sec^2\theta - 1}\, \sec\theta\tan\theta \, d\theta$$

$$= \int \tan^2\theta \sec\theta \, d\theta = \int \tan\theta \cdot \sec\theta\tan\theta \, d\theta$$

$$\left(\text{Let } u = \tan\theta, \quad dv = \sec\theta\tan\theta \, d\theta \right.$$

$$\left. du = \sec^2\theta \, d\theta, \quad v = \sec\theta \right)$$

$$= \sec\theta\tan\theta - \int \sec\theta \sec^2\theta \, d\theta$$

$$= \sec\theta\tan\theta - \int \sec\theta\,(1+\tan^2\theta)\, d$$

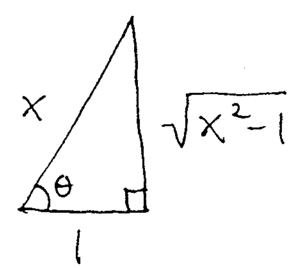
$$= \sec\theta\tan\theta - \int \sec\theta \, d\theta - \int \tan^2\theta \sec\theta \, d\theta \rightarrow$$

$$2\int \tan^2\theta \cdot \sec\theta \, d\theta = \sec\theta\tan\theta - \ln|\sec\theta+\tan\theta| + C \rightarrow$$

$$\int \tan^2\theta \sec\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta - \frac{1}{2}\ln|\sec\theta+\tan\theta| + C$$ so

$$\int\sqrt{x^2-1}\,dx = \frac{1}{2}x\cdot\sqrt{x^2-1}$$

$$-\frac{1}{2}\ln\left|x+\sqrt{x^2-1}\right| + C$$



f.) $$\int x\left(x^2-9\right)^{1/2}dx = \frac{2}{3}\left(\frac{1}{2}\right)\left(x^2-9\right)^{3/2} + C$$

g.) $\int \sec\theta \, d\theta = \int \sec\theta \, \dfrac{\sec\theta + \tan\theta}{\sec\theta + \tan\theta} \, d\theta$

$= \int \dfrac{\sec^2\theta + \sec\theta\tan\theta}{\sec\theta + \tan\theta} \, d\theta$ (Let $u = \sec\theta + \tan\theta$

$\rightarrow du = (\sec\theta\tan\theta + \sec^2\theta)\,d\theta$)

$= \int \frac{1}{u} \, du = \ln|u| + C = \ln|\sec\theta + \tan\theta| + C$

h.) $\int \sec^3\theta \, d\theta = \int \sec\theta \cdot \sec^2\theta \, d\theta$

(Let $u = \sec\theta$, $dv = \sec^2\theta \, d\theta$

$du = \sec\theta\tan\theta \, d\theta$, $v = \tan\theta$)

$= \sec\theta\tan\theta - \int \sec\theta\tan^2\theta \, d\theta$

$= \sec\theta\tan\theta - \int \sec\theta(\sec^2\theta - 1) \, d\theta$

$= \sec\theta\tan\theta - \int \sec^3\theta \, d\theta + \int \sec\theta \, d\theta$, i.e.,

$\int \sec^3\theta \, d\theta = \sec\theta\tan\theta - \int \sec^3\theta \, d\theta + \ln|\sec\theta + \tan\theta| + C \rightarrow$

$2\int \sec^3\theta \, d\theta = \sec\theta\tan\theta + \ln|\sec\theta + \tan\theta| + C \rightarrow$

$\int \sec^3\theta \, d\theta = \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta| + C$.

i.) $\int \sec^5\theta \, d\theta = \int \sec^3\theta \cdot \sec^2\theta \, d\theta$

(Let $u = \sec^3\theta$, $dv = \sec^2\theta \, d\theta$

$du = 3\sec^2\theta \cdot \sec\theta\tan\theta \, d\theta$

$= 3\sec^3\theta \tan d\theta$, $v = \tan\theta$)

$= \sec^3\theta\tan\theta - 3\int \sec^3\theta\tan^2\theta \, d\theta$

$= \sec^3\theta\tan\theta - 3\int \sec^3\theta(\sec^2\theta - 1) \, d\theta$

$= \sec^3\theta\tan\theta - 3\int (\sec^5\theta - \sec^3\theta) \, d\theta$

$= \sec^3\theta\tan\theta - 3\int \sec^5\theta \, d\theta + 3\int \sec^3\theta \, d\theta$

$= \sec^3\theta \tan\theta - 3\int \sec^5\theta \, d\theta$
$\quad + 3\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta + \tan\theta|\right] + c$, i.e.,

$\int \sec^5\theta \, d\theta = \sec^3\theta \tan\theta - 3\int \sec^5\theta \, d\theta$
$\quad + \frac{3}{2}\sec\theta\tan\theta + \frac{3}{2}\ln|\sec\theta + \tan\theta| + c \longrightarrow$

$4\int \sec^5\theta \, d\theta = \sec^3\theta\tan\theta + \frac{3}{2}\sec\theta\tan\theta$
$\quad + \frac{3}{2}\ln|\sec\theta + \tan\theta| + c \longrightarrow$

$\int \sec^5\theta \, d\theta = \frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{8}\sec\theta\tan\theta$
$\quad + \frac{3}{8}\ln|\sec\theta + \tan\theta| + c$.

j.) $\int x^2\sqrt{x^2-9}\, dx$ (Let $x = 3\sec\theta \longrightarrow$
$dx = 3\sec\theta\tan\theta \, d\theta$)

$= \int 9\sec^2\theta \cdot \sqrt{9\sec^2\theta - 9} \cdot 3\sec\theta\tan\theta \, d\theta$

$= 27\int \sec^3\theta\tan\theta \sqrt{9(\sec^2\theta - 1)} \, d\theta$

$= 27\int \sec^3\theta\tan\theta \cdot 3\sqrt{\tan^2\theta} \, d\theta$

$= 81\int \sec^3\theta \cdot \tan\theta \cdot \tan\theta \, d\theta$

$= 81\int \sec^3\theta \cdot \tan^2\theta \, d\theta$

$= 81\int \sec^3\theta \cdot (\sec^2\theta - 1) \, d\theta$

$= 81\int (\sec^5\theta - \sec^3\theta) \, d\theta$

$= 81\int \sec^5\theta \, d\theta - 81\int \sec^3\theta \, d\theta$

$= 81\left[\frac{1}{4}\sec^3\theta\tan\theta + \frac{3}{8}\sec\theta\tan\theta\right.$
$\quad \left. + \frac{3}{8}\ln|\sec\theta + \tan\theta|\right] -$

$$81\left[\frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta|\right] + C$$

$$= \frac{81}{4}\sec^3\theta\tan\theta + \frac{243}{8}\sec\theta\tan\theta$$

$$+ \frac{243}{8}\ln|\sec\theta+\tan\theta| - \frac{81}{2}\sec\theta\tan\theta$$

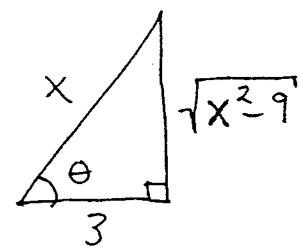
$$- \frac{81}{2}\ln|\sec\theta+\tan\theta| + C$$

$$= \frac{81}{4}\sec^3\theta\tan\theta - \frac{81}{8}\sec\theta\tan\theta$$

$$- \frac{81}{8}\ln|\sec\theta+\tan\theta| + C$$

$$= \frac{81}{4}\left(\frac{x}{3}\right)^3 \cdot \frac{\sqrt{x^2-9}}{3} - \frac{81}{8}\left(\frac{x}{3}\right)\cdot\frac{\sqrt{x^2-9}}{3}$$

$$- \frac{81}{8}\ln\left|\frac{x}{3} + \frac{\sqrt{x^2-9}}{3}\right| + C$$



k.) Let $x = \tan\theta \rightarrow dx = \sec^2\theta\, d\theta \longrightarrow$

$$\int\sqrt{x^2+1}\,dx = \int\sqrt{\tan^2\theta+1}\cdot\sec^2\theta\,d\theta = \underline{\int\sec^3\theta\,d\theta}$$

$$= \int\sec^2\theta\sec\theta\,d\theta \qquad (\text{Let } u=\sec\theta,\ dv=\sec^2\theta\,d\theta,$$
$$du = \sec\theta\tan\theta\,d\theta,\quad v=\tan\theta)$$

$$= \sec\theta\tan\theta - \int\sec\theta\cdot\tan^2\theta\,d\theta$$

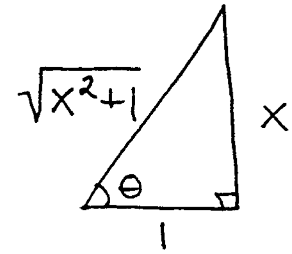
$$= \sec\theta\tan\theta - \int\sec\theta\,(\sec^2\theta-1)\,d\theta$$

$$= \sec\theta\tan\theta + \ln|\sec\theta+\tan\theta| - \underline{\int\sec^3\theta\,d\theta}$$

$$= \frac{1}{2}\sec\theta\tan\theta + \frac{1}{2}\ln|\sec\theta+\tan\theta| + C$$

$$= \frac{1}{2}\sqrt{x^2+1}\,(x)$$

$$- \frac{1}{2}\ln\left|\sqrt{x^2+1} + x\right| + c$$



l.) $\displaystyle\int \frac{\sqrt{4x^2+1}}{x}\,dx$ (Let $u^2 = 4x^2+1 \rightarrow$
$2u\,du = 8x\,dx \rightarrow$
$\frac{1}{4}u\,du = x\,dx$ and
$x^2 = \frac{1}{4}(u^2-1)$)

$$= \int \frac{\sqrt{4x^2+1}}{x^2}\cdot x\,dx$$

$$= \int \frac{u}{\cancel{\frac{1}{4}}(u^2-1)}\cdot \cancel{\frac{1}{4}}u\,du$$

$$= \int \frac{u^2-1+1}{u^2-1}\,du = \int \left[1 + \frac{1}{u^2-1}\right]du$$

$$= \int \left[1 + \frac{\frac{1}{2}}{u-1} + \frac{\frac{-1}{2}}{u+1}\right]du$$

$$= u + \frac{1}{2}\ln|u-1| - \frac{1}{2}\ln|u+1| + c$$

$$= \sqrt{4x^2+1} + \frac{1}{2}\ln\left|\sqrt{4x^2+1}-1\right| - \frac{1}{2}\ln\left|\sqrt{4x^2+1}-1\right| + c$$

m.) $\displaystyle\int 3x\,(4x^2+7)^{-1/2}dx = (3)\cdot\left(\frac{1}{8}\right)\cdot(2)(4x^2+7)^{1/2} + c$

n.) $\displaystyle\int \frac{x^3}{\sqrt{4x^2+7}}\,dx$ (Let $u = 4x^2+7 \rightarrow x^2 = \frac{1}{4}(u-7)$
$\rightarrow du = 8x\,dx \rightarrow \frac{1}{8}du = x\,dx$)

$$= \int \frac{x^2}{\sqrt{4x^2+7}}\cdot x\,dx = \int \frac{\frac{1}{4}(u-7)}{u^{1/2}}\cdot\frac{1}{8}\,du$$

$$= \frac{1}{32}\int\left(u^{1/2} - 7u^{-1/2}\right)du = \frac{1}{32}\left(\frac{2}{3}u^{3/2} - 7\cdot 2u^{1/2}\right) + c$$

$= \frac{1}{48}(4x^2+7)^{3/2} - \frac{7}{16}(4x^2+7)^{1/2} + c$

o.) $\int \frac{5}{3+\sqrt{x}}\,dx$ (Let $x = u^2 \rightarrow dx = 2u\,du$

$= \int \frac{5}{3+u} \cdot 2u\,du = 10 \int \frac{u}{u+3}\,du$

$= 10 \int \frac{u+3\ -3}{u+3}\,du = 10 \int \left[1 - \frac{3}{u+3}\right] du$

$= 10\,(u - 3\ln|u+3|) + c$

$= 10\,(\sqrt{x} - 3\ln|\sqrt{x}+3| + c$

p.) $\int \frac{\sqrt{x}}{\sqrt{x}-4}\,dx$ (Let $x = u^2 \rightarrow dx = 2u\,du$)

$= \int \frac{u}{u-4} \cdot 2u\,du$

$$\begin{array}{r|l} & 2u + 8 \\ u-4 & 2u^2 \\ & 2u^2 - 8u \\ \hline & 8u \\ & 8u - 32 \\ \hline & 32 \end{array}$$

$= \int \left[2u + 8 + \frac{32}{u-4}\right] du$

$= u^2 + 8u + 32\ln|u-4| + c$

$= x + 8\sqrt{x} + 32\ln|\sqrt{x}-4| + c$

q.) $\int \frac{4\sqrt{x}}{1+\sqrt{x}}\,dx$ (Let $x = u^4 \rightarrow dx = 4u^3\,du$)

$= \int \frac{u}{1+u^2} \cdot 4u^3\,du$

$$\begin{array}{r|l} & 4u^2 - 4 \\ u^2+1 & 4u^4 \\ & 4u^4 + 4u^2 \\ \hline & -4u^2 \\ & -4u^2 - 4 \\ \hline & 4 \end{array}$$

$= \int \left[4u^2 - 4 + \frac{4}{u^2+1}\right] du$

$= \frac{4}{3}u^3 - 4u + 4\arctan u + c$

$$= \frac{4}{3}(x^{1/4})^3 - 4(x^{1/4}) + 4\arctan(x^{1/4}) + c \quad .$$

r.) $\displaystyle\int \frac{\sqrt{x+1}}{x+2}\,dx$ (Let $u^2 = x+1 \rightarrow 2u\,du = dx \rightarrow$ $x = u^2 - 1$)

$$= \int \frac{u}{u^2+1} \cdot 2u\,du = 2\int \frac{u^2+1-1}{u^2+1}\,du$$

$$= 2\int \left[1 - \frac{1}{u^2+1}\right] du = 2(u - \arctan u) + c$$

$$= 2\left(\sqrt{x+1} - \arctan\sqrt{x+1}\right) + c$$

s.) $\displaystyle\int \frac{\sqrt{x}}{1+\sqrt{1+\sqrt{x}}}\,dx$ (Let $u^2 = 1+\sqrt{x} \rightarrow \sqrt{x} = u^2 - 1 \rightarrow$ $2u\,du = \frac{1}{2\sqrt{x}}\,dx \rightarrow 4u(u^2-1)\,du = dx$)

$$= \int \frac{u^2-1}{1+u} \cdot 4u(u^2-1)\,du$$

$$= 4\int (u^4 - u^3 - u^2 + u)\,du$$

$$= 4\left(\frac{1}{5}u^5 - \frac{1}{4}u^4 - \frac{1}{3}u^3 + \frac{1}{2}u^2\right) + c$$

$$= \frac{4}{5}\left(\sqrt{1+\sqrt{x}}\right)^5 - (1+\sqrt{x})^2 - \frac{4}{3}\left(\sqrt{1+\sqrt{x}}\right)^3 + 2(1+\sqrt{x}) + c$$

2.) a.)

$$\begin{array}{ccccc} 0 & \tfrac{1}{2} & 1 & \tfrac{3}{2} & 2 \end{array} \qquad f(x) = \frac{x}{1+x^3}$$

$$T_4 = \frac{2-0}{2(4)}\left[f(0) + 2f\left(\tfrac{1}{2}\right) + 2f(1) + 2f\left(\tfrac{3}{2}\right) + f(2)\right]$$

$$= \frac{1}{4}\left[0 + 2\left(\frac{4}{9}\right) + 2\left(\frac{1}{2}\right) + 2\left(\frac{12}{35}\right) + \frac{2}{9}\right] \approx \boxed{0.6992}$$

b.) $$S_4 = \frac{2-0}{3(4)}\left[f(0) + 4f\left(\tfrac{1}{2}\right) + 2f(1) + 4f\left(\tfrac{3}{2}\right) + f(2)\right]$$

$$\approx \boxed{0.7286}$$

c.) $$\int \frac{x}{1+x^3}\,dx = \int \frac{x}{(x+1)(x^2-x+1)}\,dx$$

$$= \int \left[\frac{A}{x+1} + \frac{Bx+C}{x^2-x+1}\right] dx = \int \left[\frac{\frac{-1}{3}}{x+1} + \frac{\frac{1}{3}x + \frac{1}{3}}{x^2-x+1}\right] dx$$

$$= \frac{-1}{3}\ln|x+1| + \frac{1}{3}\cdot\frac{1}{2}\int \frac{2x-1+3}{x^2-x+1}\,dx$$

$$= \frac{-1}{3}\ln|x+1| + \frac{1}{6}\int\left[\frac{2x-1}{x^2-x+1} + \frac{3}{\left(x-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}\right] dx$$

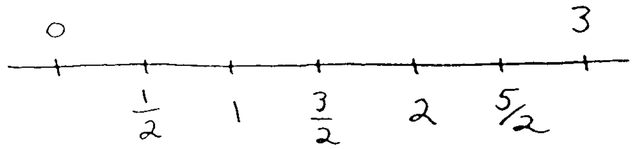
$$= \frac{-1}{3}\ln|x+1| + \frac{1}{6}\ln|x^2-x+1| + \frac{1}{2}\cdot\frac{2}{\sqrt{3}}\arctan\left(\frac{x-\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) + C$$

so

$$\int_0^2 \frac{x}{1+x^3}\,dx = \left(\frac{-1}{3}\ln 3 + \frac{1}{6}\ln 3 + \frac{1}{\sqrt{3}}\arctan\sqrt{3}\right)$$

$$- \left(\frac{1}{\sqrt{3}}\arctan\left(\frac{-1}{\sqrt{3}}\right)\right) \approx \boxed{0.7238}.$$

3.) a.)

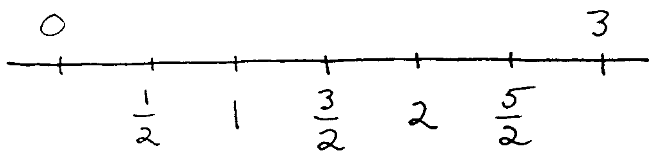


$f(x) = \frac{x}{1+x^4}$

$$T_6 = \frac{3-0}{2(6)}\left[f(0) + 2f\left(\tfrac{1}{2}\right) + 2f(1) + 2f\left(\tfrac{3}{2}\right) + 2f(2) + 2f\left(\tfrac{5}{2}\right) + f(3)\right]$$

$$= \frac{1}{4}\left[0 + \frac{16}{17} + 1 + \frac{48}{97} + \frac{4}{17} + \frac{80}{641} + \frac{3}{82}\right] \approx .70817$$

b.)



$f(x) = \frac{x}{1+x^4}$

$$S_6 = \frac{3-0}{3(6)}\left[f(0) + 4f\left(\tfrac{1}{2}\right) + 2f(1) + 4f\left(\tfrac{3}{2}\right) + 2f(2) + 4f\left(\tfrac{5}{2}\right) + f(3)\right]$$

$$= \frac{1}{6}\left[0 + \frac{32}{17} + 1 + \frac{96}{97} + \frac{4}{17} + \frac{160}{641} + \frac{3}{82}\right] \approx .73225$$

c.) $$\int_0^3 \frac{x}{1+(x^2)^2}\,dx = \frac{1}{2}\arctan(x^2)\Big|_0^3$$

$$= \frac{1}{2}\arctan 9 - \frac{1}{2}\arctan 0 \approx .73007$$

4.) a.) $$f''(x) = e^{-x^2}(4x^2 - 2)$$

$$f^{(4)}(x) = e^{-x^2}(16x^4 - 48x^2 + 12)$$

b.) $$M_2 = \max_{0 \le x \le 1} |f''(x)| \le e^0 \cdot |4-2| = 2$$

$$M_4 = \max_{0 \le x \le 1} |f^{(4)}(x)| \le e^0 |16 - 48 + 12| = 20$$

c.) i.) $\text{Err} = \dfrac{(b-a)M_2 h^2}{12} = \dfrac{1}{6n^2} < .00001 \Rightarrow$

$n \geq 121$.

ii.) $\text{Err} = \dfrac{(b-a)M_4 h^4}{180} = \dfrac{1}{9n^4} < .00001 \Rightarrow$

$n \geq 11$.

5.) $N = ce^{kt}$ and $t=0$, $N = 1000$ insects $\Rightarrow$

$N = 1000\,e^{kt}$ and $t = 3$, $N = 1250$ insects $\Rightarrow$

$1250 = 1000\,e^{3k} \Rightarrow 1.25 = e^{3k} \Rightarrow \ln 1.25 = 3k \Rightarrow$

$k = \frac{1}{3}\ln 1.25 = \ln(1.25)^{\frac{1}{3}} \Rightarrow N = 1000\left(e^{\ln(1.25)^{\frac{1}{3}}}\right)^t$ or

$\boxed{N = 1000\,(1.25)^{t/3}}$.

a.) $t = 14$ days $\Rightarrow$ $N = 2833$ insects

b.) $N = 10{,}000$ insects $\Rightarrow$

$10{,}000 = 1000\,(1.25)^{t/3} \Rightarrow 10 = (1.25)^{t/3} \Rightarrow$

$\ln 10 = \frac{t}{3}\ln(1.25) \Rightarrow t = \dfrac{3\ln 10}{\ln(1.25)} \approx 31$ days.

6.) N: # of elk $\quad t$: # of years

$\frac{dN}{dt} = kN \Rightarrow \int \frac{1}{N}\,dN = \int k\,dt \Rightarrow \ln N = kt + c \Rightarrow$

$N = e^{kt+c} = e^c e^{kt} = ce^{kt}$ or $N = ce^{kt}$.

When $t=0$ (1988), $N = 300 \Rightarrow N = 300\,e^{kt}$ and

when $t = 1$ (1989), $N = 336 \Rightarrow 336 = 300\,e^{k} \Rightarrow$

$1.12 = e^k \Rightarrow \ln 1.12 = k \Rightarrow N = 300\left(e^{(\ln 1.12)}\right)^t = 300(1.12)^t$ or

$\boxed{N = 300\,(1.12)^t}$. When $t = 5$ (1993), $N \approx 529$ elk.

7.) W: wt. of fungus (oz.)   t: # of days

$$\frac{dW}{dt} = kW^2 \Rightarrow \int \frac{1}{W^2}\,dW = \int k\,dt \Rightarrow \frac{-1}{W} = kt + c \Rightarrow$$

$$W = \frac{-1}{kt+c} \quad \text{and} \quad t=0,\ W = 3 \text{ oz.} \Rightarrow 3 = \frac{-1}{c} \Rightarrow c = \frac{-1}{3} \Rightarrow$$

$$W = \frac{-1}{kt - \frac{1}{3}} = \frac{3}{1-3kt}. \quad \text{also, } t=4,\ W = 5 \text{ oz.} \Rightarrow$$

$$5 = \frac{3}{1-12k} \Rightarrow k = \frac{1}{30} \Rightarrow \boxed{W = \frac{3}{1-.1t}}.$$

a.) $t = 9 \Rightarrow W = 30$ oz.

b.) $W = 48$ oz. $\Rightarrow 48 = \dfrac{3}{1-.1t} \Rightarrow 48 - 4.8t = 3 \Rightarrow$

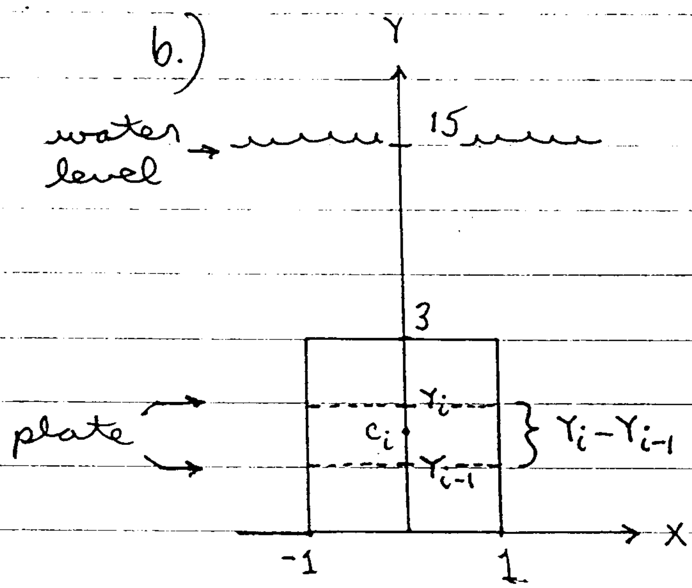
$t = 9.375$ days $= 9$ days and 9 hours.

8.) a.) Force $=$ (area) $\times$ (depth) $\times$ (specific wt.)

$$= (\pi(5)^2) \times (100) \times (62.4) = 156{,}000\pi$$

$$= 490{,}088.5 \text{ lbs.}$$

b.)



Divide the interval $[0,3]$ on the Y-axis into n parts and let c_i be the sampling point between Y_{i-1} and Y_i.

Force on "slice" is estimated to be

$$\underbrace{2(Y_i - Y_{i-1})}_{\text{area}} \times \underbrace{(15 - c_i)}_{\text{depth}} \times \underbrace{(62.4)}_{\text{specific wt.}}$$

so exact force is

$$\text{Force} = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} 2(Y_i - Y_{i-1})(15 - c_i)(62.4)$$

$$= \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} (124.8)(15 - c_i) \cdot (Y_i - Y_{i-1})$$

$$= \int_0^3 (124.8)(15 - Y)\, dy$$

$$= (124.8)\left(15Y - \frac{Y^2}{2}\right)\Big|_0^3 = 5054.4 \text{ lbs.}$$

9.) a.) $f'(x) = x^2 + 1 \Rightarrow f(x) = \frac{x^3}{3} + x + c$

b.) $\frac{dY}{dx} = e^x Y \Rightarrow \int \frac{1}{Y}\, dy = \int e^x dx \Rightarrow$

$\ln Y = e^x + c \Rightarrow Y = e^{e^x + c} = e^c e^{e^x} = c e^{e^x}$ or

$Y = c e^{e^x}$.

c.) $\frac{dY}{dx} = Y^2 \Rightarrow \int \frac{1}{Y^2}\, dy = \int dx \Rightarrow$

$\frac{-1}{Y} = x + c \Rightarrow Y = \frac{-1}{x+c}$.

d.) $\frac{dY}{dx} = \frac{Y^2+1}{2Y} \Rightarrow \int \frac{2Y}{Y^2+1}\, dy = \int dx \Rightarrow$

$\ln(Y^2+1) = x + c \Rightarrow Y^2 + 1 = e^{x+c} = e^c e^x = c e^x \Rightarrow$

$Y^2 = c e^x - 1 \Rightarrow Y = \pm\sqrt{c e^x - 1}$.