ESP
Kouba
Worksheet 17

1.) Graph the curve represented by the following pairs of parametric equations. If possible, eliminate t and write an equation for the curve in rectangular coordinates.

a.) $x = t - 1 , \ y = t + 1$
b.) $x = t , \ y = t^2$
c.) $x = t^2 , \ y = t^4$
d.) $x = e^t , \ y = e^{2t}$
e.) $x = \cos t , \ y = \sin t$
f.) $x = 3 \cos t , \ y = \sin t$
g.) $x = t^2 - t , \ y = t^2$
h.) $x = \ln t , \ y = t + 1/t$

2.) Determine the slope of the line tangent to the following graphs at the indicated value.

a.) $y = (\pi - \arctan x)^4$ at $x = 1$
b.) $x = t^2 + 1 , \ y = e^{-t} + t$ at $t = 1$
c.) $r = 3 + \sin \theta$ at $\theta = \pi/4$

3.) Compute dy/dx and $d^2 y / dx^2$ for each of the following.

a.) $y = x / (x^2 + 1)$
b.) $x = t + \sin t , \ y = e^{\tan t} - t$
c.) $r = \theta$
d.) $r = \sin \theta$

4.) Consider the curve given parametrically by

$$x = t^2 + e^t \text{ and } y = t + e^t \text{ for } t \text{ in } [0, 1] .$$

Find the area of the region lying under the curve and above the x-axis for x in $[1, 1 + e]$.

5.) Compute the arc lengths of the given curves over the indicated intervals.

a.) $y = x^{5/4}$ for x in $[0, 1]$
b.) $y = 1/(2x^2) + x^4/16$ for x in $[2, 3]$
c.) $x = \cos t + t \sin t$ and $y = \sin t - t \cos t$ for t in $[\pi/6, \pi/4]$
d.) $r = \sin^2(\theta/2)$ for θ in $[0, \pi]$

6.) Consider a particle moving along the curve given parametrically by

$$x = t + \cos t \quad \text{and} \quad y = t - \sin t \quad \text{for } t \geq 0 .$$

a.) Determine a formula for the speed (ft./sec.) of the particle at time t.
b.) What is the speed when $t = 0$ sec. ? $t = \pi/2$ sec. ? $t = 100$ sec. ?

7.) Compute the *curvature* of the given curve at the given point.

a.) $y = x^3$ at $(-1, 1)$
b.) $y = e^{x^2}$ at $(1, e)$
c.) $x = t^2 - t$, $y = t^2 + t$ at $t = 1$