ESP
Kouba
Worksheet 2 Solutions

1.) a.) $\displaystyle\sum_{i=1}^{300} \pi = 300\pi$

b.) $\displaystyle\sum_{i=1}^{200} (3i-2) = 3\left(\sum_{i=1}^{200} i\right) - \sum_{i=1}^{200} 2$

$\displaystyle = 3 \cdot \frac{200(200+1)}{2} - 200 \cdot 2 = 59{,}900 .$

c.) $\displaystyle\sum_{i=1}^{500} (5i^2 - i + 1) = 5\left(\sum_{i=1}^{500} i^2\right) - \sum_{i=1}^{500} i + \sum_{i=1}^{500} 1$

$\displaystyle = 5 \cdot \frac{500(500+1)(1000+1)}{6} - \frac{500(500+1)}{2} + 500 \cdot 1 = 208{,}834{,}000 .$

d.) $\displaystyle\sum_{i=1}^{1000} (i^2 + i) = \sum_{i=1}^{1000} i^2 + \sum_{i=1}^{1000} i$

$\displaystyle = \frac{1000(1000+1)(2000+1)}{6} + \frac{1000(1000+1)}{2} = 334{,}334{,}000 .$

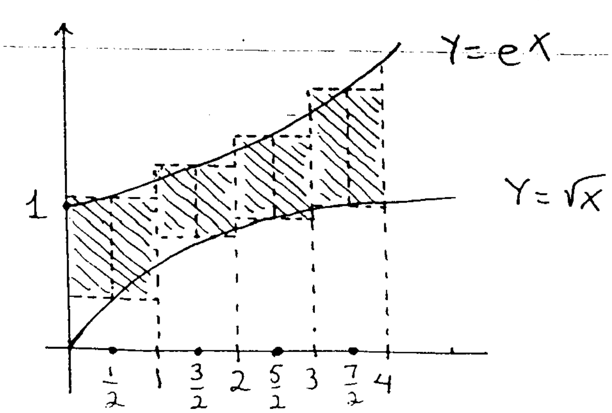
e.) $\displaystyle\sum_{i=253}^{684} (i-1)^2 = \sum_{i=252}^{683} i^2 = \sum_{i=1}^{683} i^2 - \sum_{i=1}^{251} i^2$

$\displaystyle = \frac{683(683+1)(1366+1)}{6} - \frac{251(251+1)(502+1)}{6} = 101{,}134{,}728 .$

f.) $\displaystyle\sum_{i=1}^{203} \left[(i+1)^3 - i^3\right] = (\cancel{2^3} - 1^3) + (\cancel{3^3} - \cancel{2^3}) + (\cancel{4^3} - \cancel{3^3}) + (\cancel{5^3} - \cancel{4^3}) + \cdots$

$+ (\cancel{203^3} - \cancel{202^3}) + (204^3 - \cancel{203^3}) = 204^3 - 1^3 = 8{,}489{,}663 .$

2.)

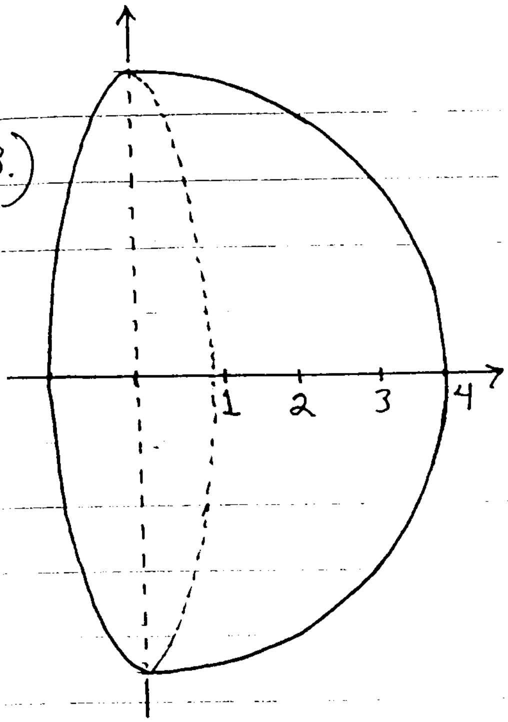


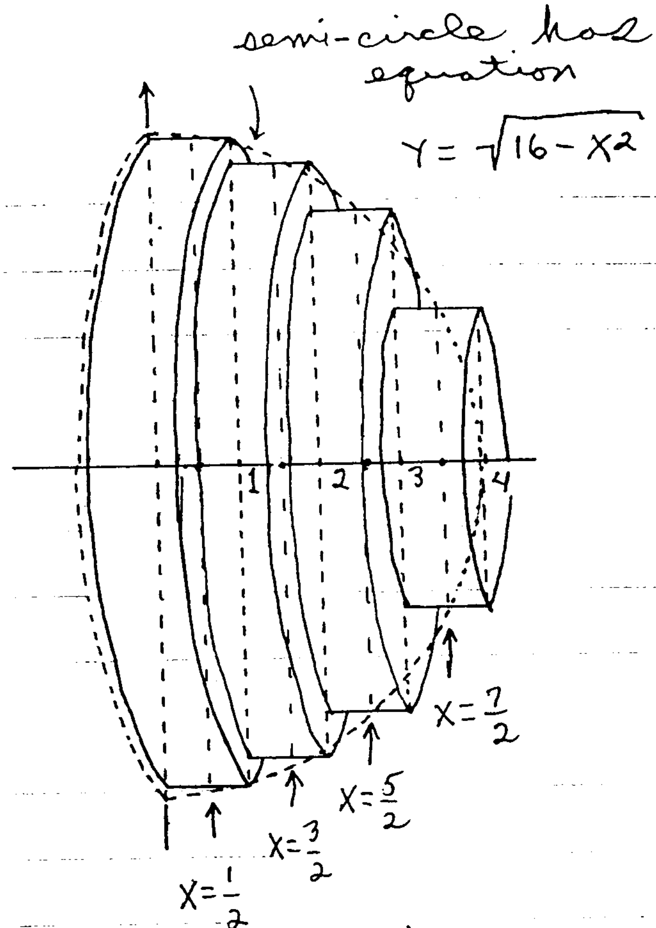
$\displaystyle \text{Area} \approx \left(e^{1/2} - \sqrt{\tfrac{1}{2}}\right)\cdot 1 + \left(e^{3/2} - \sqrt{\tfrac{3}{2}}\right)\cdot 1$

$\displaystyle + \left(e^{5/2} - \sqrt{\tfrac{5}{2}}\right)\cdot 1 + \left(e^{7/2} - \sqrt{\tfrac{7}{2}}\right)\cdot 1$

≈ 46.045

3.)

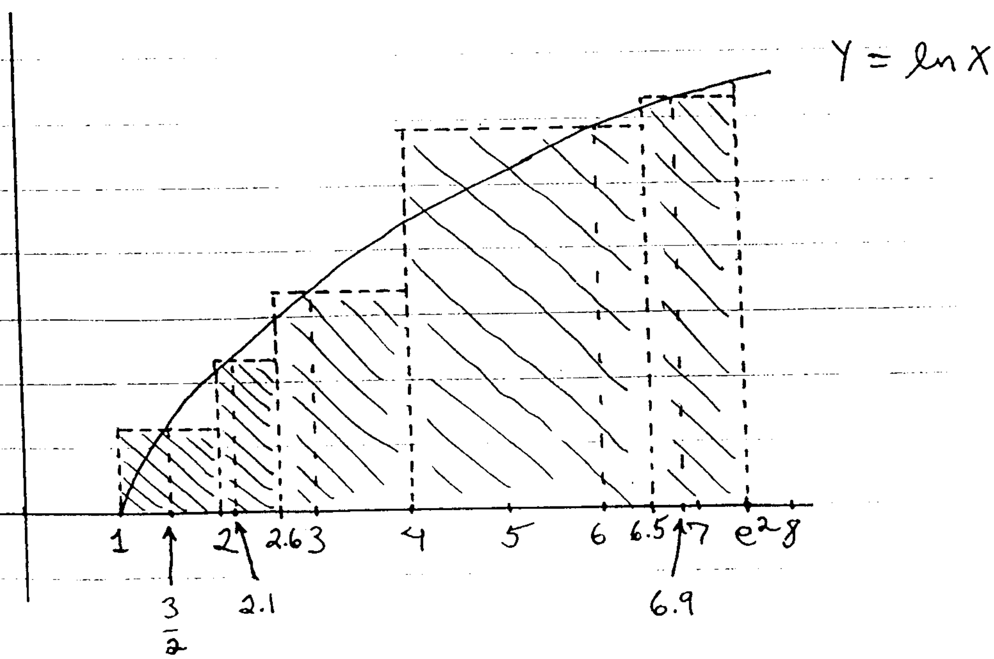




$$\text{Volume} \approx \pi\left(\frac{63}{4}\right)\cdot 1 + \pi\left(\frac{55}{4}\right)\cdot 1 + \pi\left(\frac{39}{4}\right)\cdot 1 + \pi\left(\frac{15}{4}\right)\cdot 1$$

$$= 43\pi \approx 135.09$$

4.)



$$\text{Area} \approx \left(\ln\frac{3}{2}\right)(1) + (\ln 2.1)(.6) + (\ln 3)(1.4) + (\ln 6)(2.5) + (\ln 6.9)(e^2 - 6.5)$$

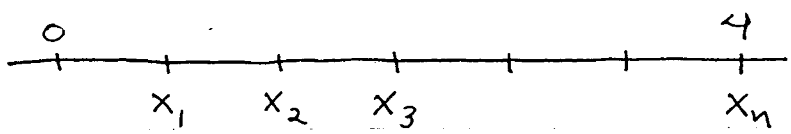
$$\approx 8.585$$

5.) a.) $(\ln 2)(2) + (\ln 4.5)(3) + \left(\ln\left(\frac{6+e^2}{2}\right)\right)(e^2-6) \approx 8.54$

b.) $e^{(\frac{-1}{2})^2}\cdot\left(\frac{1}{2}\right) + e^{(0)^2}\cdot\left(\frac{1}{2}\right) + e^{(\frac{1}{2})^2}\cdot\left(\frac{1}{2}\right) + e^{(1)^2}\cdot\left(\frac{1}{2}\right) \approx 3.143$

c.) $\tan\left(\frac{-\pi}{4}\right)\cdot\frac{\pi}{12} + \tan\left(\frac{-\pi}{6}\right)\cdot\frac{\pi}{12} + \tan\left(\frac{-\pi}{12}\right)\cdot\frac{\pi}{12} \approx -.483$

6.)



Divide $[0,4]$ into n equal parts, each of length $\frac{4}{n}$. Then formula for x_i is $x_i = \frac{4i}{n}$. Thus,

$$\text{Area} \approx \sum_{i=1}^{n} f(x_i)\cdot\frac{4}{n} = \sum_{i=1}^{n} f\left(\frac{4i}{n}\right)\cdot\frac{4}{n}$$

$$= \sum_{i=1}^{n}\left[\frac{1}{2}\left(\frac{4i}{n}\right)^2 + \left(\frac{4i}{n}\right)\right]\cdot\frac{4}{n}$$

$$= \sum_{i=1}^{n}\left(\frac{32}{n^3}i^2 + \frac{16}{n^2}i\right) = \frac{32}{n^3}\left(\sum_{i=1}^{n} i^2\right) + \frac{16}{n^2}\left(\sum_{i=1}^{n} i\right)$$

$$= \frac{32}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} + \frac{16}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= \frac{16}{3}\left(\frac{2n^2+3n+1}{n^2}\right) + 8\left(\frac{n+1}{n}\right) = \frac{16}{3}\left(2+\frac{3}{n}+\frac{1}{n^2}\right) + 8\left(1+\frac{1}{n}\right)$$

a.) If $n=2$, then Area ≈ 32

b.) If $n=4$, then Area ≈ 25

c.) If $n=20$, then Area ≈ 19.88

d.) If $n=100$, then Area ≈ 18.9072

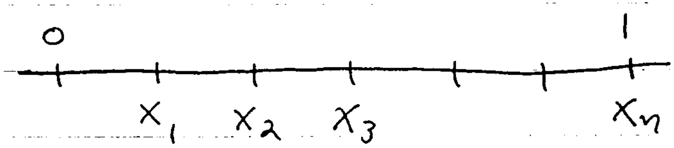
e.) $\lim\limits_{n\to\infty}\left(\frac{16}{3}\left(2+\frac{3}{n}+\frac{1}{n^2}\right)+8\left(1+\frac{1}{n}\right)\right)=\frac{32}{3}+8=18\frac{2}{3}$.

7.) a.) mesh $=\frac{2}{10}=\frac{1}{5}$

b.) mesh $=\frac{6}{5}$

c.) mesh $=1.8$

8.) a.)

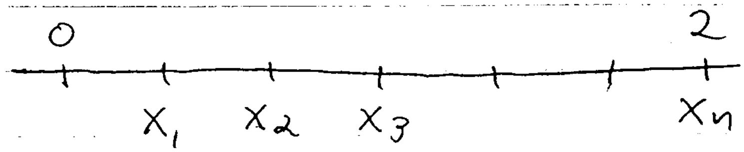


Divide $[0,1]$ into n equal parts each of length $\frac{1}{n}$ so $x_i=\frac{i}{n}$. Then for $f(x)=7$

$$\sum_{i=1}^{n} f(x_i)\cdot\frac{1}{n}=\sum_{i=1}^{n}7\cdot\frac{1}{n}=n\cdot 7\cdot\frac{1}{n}=7 \quad \text{so that}$$

$$\int_0^1 7\,dx=\lim_{n\to\infty}\sum_{i=1}^{n}f(x_i)\cdot\frac{1}{n}=\lim_{n\to\infty}7=7.$$

b.)



Divide $[0,2]$ into n equal parts each of length $\frac{2}{n}$ so $x_i=\frac{2i}{n}$. Then for $f(x)=3x-1$

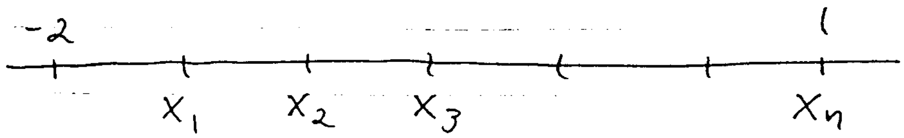
$$\sum_{i=1}^{n} f(x_i)\cdot\frac{2}{n}=\sum_{i=1}^{n}\left[3\left(\frac{2i}{n}\right)-1\right]\cdot\frac{2}{n}=\sum_{i=1}^{n}\left[\frac{12}{n^2}i-\frac{2}{n}\right]$$

$$=\frac{12}{n^2}\left(\sum_{i=1}^{n}i\right)-\sum_{i=1}^{n}\frac{2}{n}=\frac{12}{n^2}\cdot\frac{n(n+1)}{2}-n\cdot\frac{2}{n}$$

$= 6\left(1+\frac{1}{n}\right) - 2$ so that

$$\int_0^2 (3x-1)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\frac{2}{n}$$

$$= \lim_{n\to\infty} \left\{6\left(1+\frac{1}{n}\right)-2\right\} = 4 .$$

c.) 

Divide

$[-2,1]$ into n equal parts each of length $\frac{3}{n}$ so $x_i = -2 + \frac{3i}{n}$. Then for $f(x) = x^2 + x$

$$\sum_{i=1}^{n} f(x_i)\cdot\frac{3}{n} = \sum_{i=1}^{n} f\left(-2+\frac{3i}{n}\right)\cdot\frac{3}{n} = \sum_{i=1}^{n}\left[\left(-2+\frac{3i}{n}\right)^2+\left(-2+\frac{3i}{n}\right)\right]\cdot\frac{3}{n}$$

$$= \sum_{i=1}^{n}\left[4-\frac{12i}{n}+\frac{9i^2}{n^2}-2+\frac{3i}{n}\right]\cdot\frac{3}{n} = \sum_{i=1}^{n}\left[\frac{9i^2}{n^2}-\frac{9i}{n}+2\right]\cdot\frac{3}{n}$$

$$= \frac{27}{n^3}\left(\sum_{i=1}^{n} i^2\right) - \frac{27}{n^2}\left(\sum_{i=1}^{n} i\right) + \left(\sum_{i=1}^{n}\frac{6}{n}\right)$$

$$= \frac{27}{n^3}\cdot\frac{n(n+1)(2n+1)}{6} - \frac{27}{n^2}\cdot\frac{n(n+1)}{2} + n\cdot\frac{6}{n}$$

$$= \frac{9}{2}\left(2+\frac{3}{n}+\frac{1}{n^2}\right) - \frac{27}{2}\left(1+\frac{1}{n}\right) + 6$$ so that

$$\int_{-2}^{1} (x^2+x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\cdot\frac{3}{n}$$

$$= \frac{18}{2} - \frac{27}{2} + 6$$

$$= 3/2$$