ESP

Kouba

Worksheet 4 Solutions

1.) a.) $\int_0^3 7\,dx = 7x\Big|_0^3 = 21-0 = 21$

b.) $\int_{-1}^2 (x^2+x)\,dx = \left(\frac{1}{3}x^3+\frac{1}{2}x^2\right)\Big|_{-1}^2 = \frac{9}{2}$

c.) $\int_2^3 x(x+7)^2\,dx = \int_2^3 x(x^2+14x+49)\,dx$

$= \int_2^3 (x^3+14x^2+49x)\,dx = \left(\frac{1}{4}x^4+\frac{14}{3}x^3+\frac{49}{2}x^2\right)\Big|_2^3$

$= \frac{2729}{12}$

d.) $\int_0^1 \frac{x^2+5x+6}{x+3}\,dx = \int_0^1 \frac{(x+2)\cancel{(x+3)}}{\cancel{x+3}}\,dx = \int_0^1 (x+2)\,dx$

$= \left(\frac{1}{2}x^2+2x\right)\Big|_0^1 = \frac{5}{2}$

e.) $\int_1^e \frac{x^2-1+x^{-3}}{x^2}\,dx = \int_1^e (1-x^{-2}+x^{-5})\,dx$

$= \left(x+x^{-1}+\frac{x^{-4}}{-4}\right)\Big|_1^e = \left(x+\frac{1}{x}-\frac{1}{4x^4}\right)\Big|_1^e = e+\frac{1}{e}-\frac{1}{4e^4}-\frac{7}{4}$

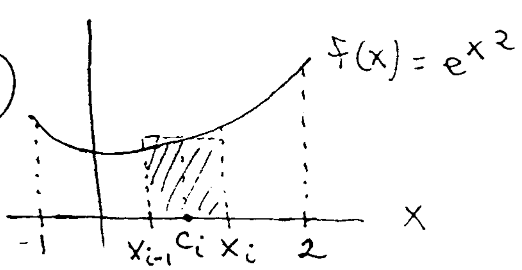
f.) $\int_0^{\frac{\pi}{2}} \cos 3x\,dx = \frac{1}{3}\sin 3x\Big|_0^{\frac{\pi}{2}}$

$= \frac{1}{3}\sin\frac{3\pi}{2} - \frac{1}{3}\sin 0 = \frac{-1}{3}$

g.) $\int_0^1 x\cdot(1+x^2)^{1/2}\,dx = \frac{1}{2}\cdot\frac{2}{3}(1+x^2)^{3/2}\Big|_0^1$

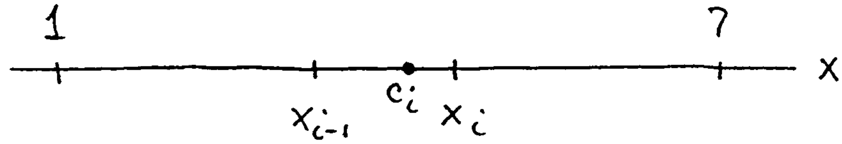
$= \frac{1}{3}(1+x^2)^{3/2}\Big|_0^1 = \frac{1}{3}\cdot 2^{3/2}-\frac{1}{3}\cdot 1 = \frac{1}{3}(2^{3/2}-1)$

2.)



$\text{Area} = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} f(c_i)(x_i-x_{i-1})$

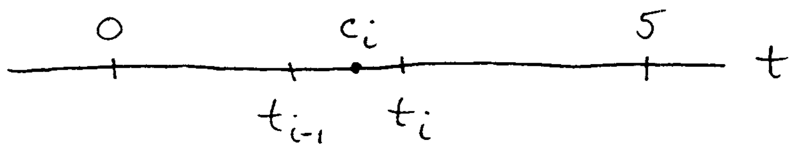
$= \int_{-1}^2 f(x)\,dx = \int_{-1}^2 e^{x^2}\,dx.$

3.) 

$f(x) = \dfrac{1}{x^2+1}$

$$\text{Mass} = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} f(c_i)\,(x_i - x_{i-1})$$

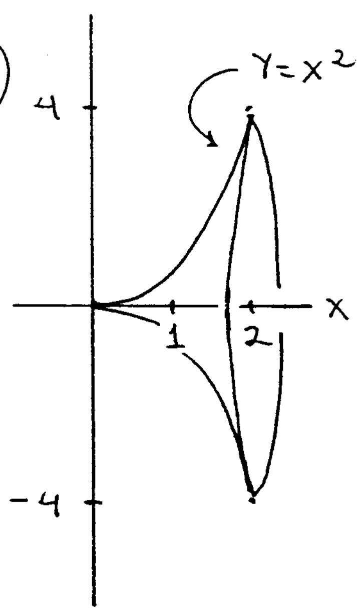
$$= \int_1^7 f(x)\,dx = \int_1^7 \frac{1}{x^2+1}\,dx .$$

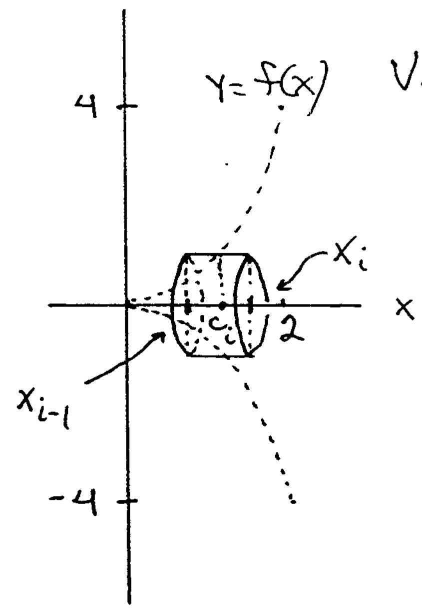
4.) 

$g(t) = 2t\sin(t^2+3)$

$$\text{Distance} = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} g(c_i)\cdot(t_i - t_{i-1})$$

$$= \int_0^5 g(t)\,dt = \int_0^5 2t\sin(t^2+3)\,dt .$$

5.) 



$$\text{Volume} = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} \pi\left[f(c_i)\right]^2 (x_i - x_{i-1})$$

$$= \int_0^2 \pi\left[f(x)\right]^2\,dx$$

$$= \int_0^2 \pi\,(x^2)^2\,dx$$

$$= \int_0^2 \pi x^4\,dx$$

6.) a.) $F'(x) = \dfrac{1}{(x^3)^2+1}\cdot 3x^2 = \dfrac{3x^2}{x^6+1}$

b.) $F'(x) = \cos(\ln(3-x))\cdot \dfrac{1}{3-x}\cdot(-1)$

c.) $F'(x) = 5\left[e^{\sin^2 x} + e^3\right]^4 \cdot e^{\sin^2 x} \cdot 2\sin x \cdot \cos x$

d.) $F'(x) = e^{x^2}$

e.) $F'(x) = e^{x^2}$

f.) If $G(x) = \int_1^x \sin(t^{20})\,dt$, then $G'(x) = \sin(x^{20})$.

Then $F(x) = G(x^3)$ so $F'(x) = G'(x^3) \cdot 3x^2$

$$= \sin\left((x^3)^{20}\right) \cdot 3x^2 = \sin(x^{60}) \cdot 3x^2$$

$= -\int_b^x f(t)\,dt$

7.) a.) $F(a) = \int_a^a f(t)\,dt = 0$

b.) $F(b) - \left(\int_a^x f(t)\,dt\right) = \left(\int_a^b f(t)\,dt\right) - \left(\int_a^x f(t)\,dt\right)$

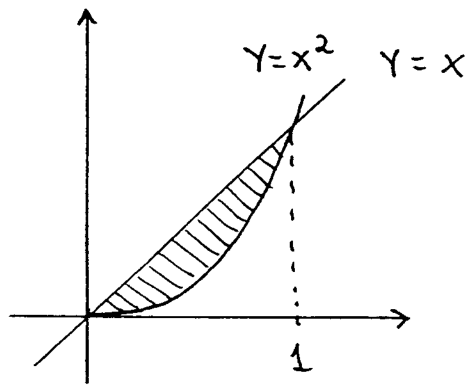
c.) Yes; since $F'(x) = f(x)$ we see that function F is differentiable. If F is differentiable, then F is continuous.

8.) Let $F(x) = \int_3^x \frac{1}{t^5+1}\,dt$ then

$$\lim_{h\to 0} \frac{\int_3^{x+h} \frac{1}{t^5+1}\,dt - \int_3^x \frac{1}{t^5+1}\,dt}{h} = \lim_{h\to 0} \frac{F(x+h) - F(x)}{h}$$

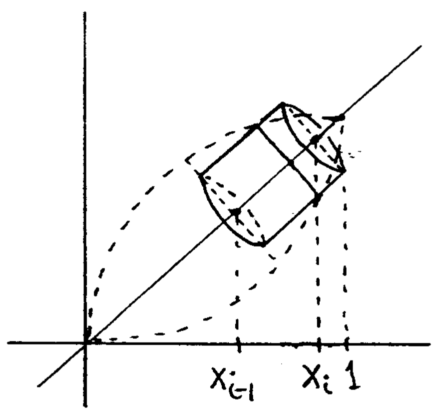
$$= F'(x) = \frac{1}{x^5+1}.$$

9.) a.)

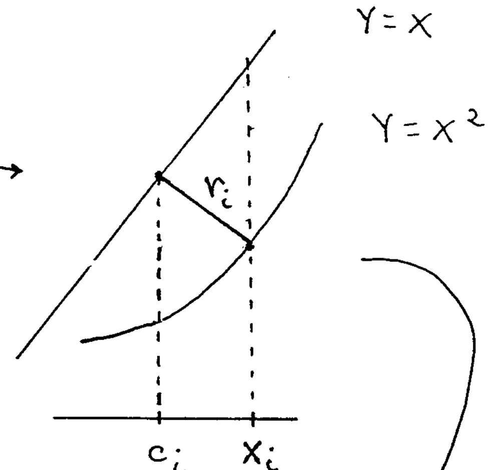


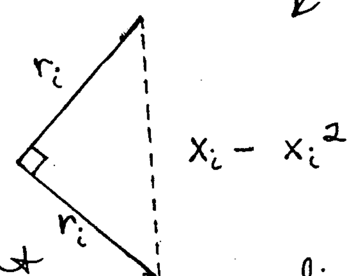
b.) i.) $\text{Volume} = \int_0^1 \pi (x)^2\,dx - \int_0^1 \pi (x^2)^2\,dx$

ii.)



radius of cylinder



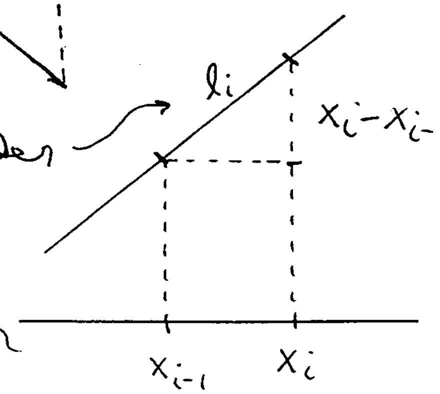


$2 \cdot (r_i)^2 = (x_i - x_i^2)^2$

and

$l_i = \sqrt{2}\,(x_i - x_{i-1})$ so

height of cylinder



$\text{Volume} = \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} \pi (r_i)^2 \cdot l_i$

$= \lim_{\text{mesh}\to 0} \sum_{i=1}^{n} \pi \cdot \frac{1}{2}(x_i - x_i^2)^2 \cdot \sqrt{2}\,(x_i - x_{i-1})$

$= \frac{1}{\sqrt{2}}\,\pi \int_0^1 (x - x^2)^2\,dx$